Math 251:10–12 — Spring 1999
MW6 CHM-106
Prof. Bumby
Lab 1 VECTOR CALCULUS

This description of the lab is only for the sections identified in the header. Other sections will use a slightly different version. However, the same lab1.mws file can be used.

Please turn in only the printout of your Maple worksheet. Use the **text** feature of Maple to add a header containing your name, identify the separate parts of the lab, labels for the plots in your worksheet and explicit answers to all questions asked (do not write this material in by hand). Remove from the worksheet any extraneous material such as the output of commands like `with(plots);` and any errors you have made. To be sure that the input and output shown agree, you should use a `restart` command and then select **Execute Worksheet** from the Edit Menu.

You can save yourself a lot of typing by first copying an already prepared Maple worksheet file to your home directory. This is done by typing `cp ~falk/maplelabs/lab1.mws .` (note the final dot which stands for "current directory") **before** starting up Maple. A copy of this file should also be available from the Web Page for this course. After you start up xmaple, open the file lab1.mws, as described in the **Instructions for Use of Maple in Mathematics 251**.

1. Execute the following sequence of commands (included in lab1.mws) which illustrate how vectors are defined in Maple, how Maple computes dot and cross products and the length of a vector, and forms the product of a scalar with a vector.

```
# Dot and Cross Products and Norm
with(linalg):  with(plots):
v:= [1,2,3];
w:= [-1,1,2];
dpvw:= dotprod(v,w);
cpvw:= crossprod(v,w);
lenv:= sqrt(dotprod(v,v));  # compute the length of v
u := v/lenv; # produces a unit vector in the direction of v
w[1];   # gives the first component of w
```

Note that some of the Maple commands described above will be needed to do Problem 2.

2a. Execute the following sequence of Maple commands (included in lab1.mws), which illustrate how vector functions are defined, differentiated, evaluated, and plotted.

```
# Vector Functions -- Space Curves
restart;
with(linalg):  with(plots):
r:= [1-4*Pi^2*(cos(t)^2-sin(t)^2),cos(t),sin(t)];
v:= diff(r,t);   # differentiates r with respect to t
subs(t=1,r);     # evaluates r at t=1
spacecurve(r,t=-Pi..2*Pi); # Plots the space curve r
```

2b. With `v` defined as in 2a, compute the length of `v` (note how this was done in Problem 1a) and use the `simplify` command to simplify it.
2c. The following formulas for the unit tangent vector T, normal vector N, and the curvature κ can be found in your book:

$$\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|}, \qquad \mathbf{N} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}, \qquad \kappa = \frac{|d\mathbf{T}/dt|}{|\mathbf{v}|}.$$

Use Maple to define expressions for each of these, using the letters given above (note that *Maple* recognizes "kappa" as the name of κ), and then simplify these expressions as much as possible. Do not `restart` since you will need the expressions defined previously in parts 2a and 2b. Note that Maple often defines intermediate quantities (such as `%1, %2`) and then writes the desired expression using these quantities.

Maple does computations in the most general case, assuming that quantities may take on complex (imaginary) values (denoted by the letter `I`) as well as real values. To get Maple to give results which are much simpler by restricting quantities to real values, you must slightly modify some of the commands. In this problem, when you compute $|d\mathbf{T}/dt|$, use the `dotprod` command of the form `dotprod(u,v,'orthogonal');` which tells Maple that the vectors `u` and `v` are real. Also note that the result obtained by Maple for κ has an obvious further simplification. Rather than spending time figuring out how to get Maple to do it, leave this answer as is and proceed to the next part.

You should experiment with different ways to arrive at these results and keep the one you like best.
2d. As a further check of your result, use Maple to compute the curvature using the formula

$$\kappa_1 = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3},$$

where $\mathbf{a}$ is the acceleration vector $\mathbf{a} = d\mathbf{v}/dt$.

Note that Maple gives you an error message if you try to apply the simplify command to the cross product of two vectors. In 2d, this can be avoided by first computing $|\mathbf{v} \times \mathbf{a}|$ before simplifying.

Use the **text** feature to compare your results for κ and κ_1. If they are not *identical*, you should indicate why they are *equal*.
2e. Use Maple to verify that the vectors T and N are orthogonal at any value of t, i.e., verify that $T \cdot N = 0$. You may need to use the `simplify` command.
2f. As a check, find the numerical value of κ at some values of t. For example, to find the value at $t = \pi/2$, use the command `evalf(subs(t=Pi/2,kappa))`. Your values should be approximately 0.00016 at $\pi/4$ and 158 at $\pi/2$. (your answer should give the normal accuracy reported by *Maple*.) Use the **text** feature to comment on how these numerical values relate to the graph obtained in 2a. (Additional experiments may help you to identify different parts of that curve, but these experiments need not be shown, although they may be mentioned in your description.)

Comment. The instructions in this lab have attempted to steer you around the difficulties in the way that *Maple* represents vectors, by representing them in a simpler form (called a "list" in the *Maple* documentation). The results of some constructions will refuse to combine in simple ways. If nothing seems to work, consult the help files for the `evalm` command for working with vectors and matrices, or the `map` command for applying functions to the elements of an array. Also note that the `with(linalg)` and `with(plots)` commands are used to make special features of *Maple* available in your worksheet.