Math 251:10–12 — Spring 1999
MW6 CHM-106
Prof. Bumby

First set of workshop problems — Vectors.
Textbook Sections 11.1 through 11.5.

You may be asked to write up *one* of these workshop problems and turn it in in recitation next week. NOTE: Although you are encouraged to work with other students on solving these problems, the writeup you turn in should represent your own presentation of the solution, not prepared in cooperation with anyone else.

Your writeup will be graded for presentation as well as mathematical correctness. You should:

1. Work out the problem and have a clear plan of presentation before you begin to write up your final solution.

2. Be neat and write legibly.

3. Show all steps. Explain what you are doing at each step in complete sentences to convince the grader that you have understood the main point of the problem. It has been suggested that you model your presentation on the way the book writes up the solution of an example, but you should really try to do better. Try to capture the adventure of new discovery that would have been lost by the time that someone writes a textbook.

1. Let C be the point on the line segment AB that is twice as far from B as it is from A. Let $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{b} = \overrightarrow{OB}$, and $\mathbf{c} = \overrightarrow{OC}$.

a) Express $\overrightarrow{AC}$ and $\overrightarrow{CB}$ in terms of $\overrightarrow{AB}$.

b) Show that $(2/3)\mathbf{a} + (1/3)\mathbf{b} = \mathbf{c}$.

c) If D is the point on the line segment AB that is three times as far from B as it is from A, find an expression for $\mathbf{d} = \overrightarrow{OD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

2. If $\mathbf{c} = |\mathbf{a}|\ \mathbf{b} + |\mathbf{b}|\ \mathbf{a}$, where $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are all nonzero vectors, show that $\mathbf{c}$ bisects the angle between $\mathbf{a}$ and $\mathbf{b}$.

3. Let P be a point not on the plane that passes through the points Q, R, and S. Show that the distance d from P to the plane is

$$d = \frac{|\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})|}{|\mathbf{a} \times \mathbf{b}|},$$

where $\mathbf{a} = \overrightarrow{QR}$, $\mathbf{b} = \overrightarrow{QS}$, and $\mathbf{c} = \overrightarrow{QP}$.