Math 251:10–12 — Spring 1999
MW6 CHM-106
Prof. Bumby

Sixth set of workshop problems — Maxima and Minima
Textbook Sections 12.7 – 12.8

1. A flat circular plate has the shape of the region $x^2+y^2 \le 1$. The plate (including the boundary $x^2+y^2 = 1$) is heated so that the temperature T at any point (x, y) is given by $T = x^3 - x + 2y^2$. Locate the hottest and coldest points of the plate and determine the temperature at each of these points.

2. You are an engineer of a packaging company. You are told to design a cylindrical can (with lid) that holds 1 liter of liquid. Naturally, you want to this in such a way that the amount of material required is a minimum. What are the dimensions (in cm) of this can? (Note: if you use too much material, *you* will be canned.)

3. Consider the function

$$Q(x, y) = 3x^2 + 8xy - 3y^2.$$

(a) Show that the only critical point of $Q(x, y)$ is $(0, 0)$ and that this point is a saddle point, so that the maximum and minimum of Q on any region will be attained on the boundary of the region.

(b) Let

$$\mathcal{D} = \Big\{ (x, y) : x^2 + y^2 \le 1 \Big\},$$

whose boundary is

$$\mathcal{C} = \Big\{ (x, y) : x^2 + y^2 = 1 \Big\}.$$

Find the maximum and minimum of Q on $\mathcal{C}$, which also gives the extreme values of Q on $\mathcal{D}$ by (a), by parameterizing $\mathcal{C}$ as

$$x = \cos t \quad y = \sin t.$$

That is, construct the function

$$q(t) = Q(\cos t, \sin t),$$

and find the maximum and minimum and minimum of $q(t)$.

(c) Use the method of *Lagrange multipliers* to find the extreme values of Q on $\mathcal{C}$.

(d) Let

$$\mathcal{B} = \Big\{ (x, y) : |x| \le 1, |y| \le 1 \Big\},$$

whose boundary $\mathcal{S}$ consists of the four line segments joining the points $(-1, -1)$, $(1, -1)$, $(1, 1)$, $(-1, 1)$, $(-1, -1)$. Find the maximum and minimum of Q on $\mathcal{S}$. You will need to consider the four vertices of the square $\mathcal{S}$ and any point on the interior of the sides of $\mathcal{S}$ (selected either by Lagrange multipliers or explicit parametrization) which is a possible location of an extreme value.