Math 251:10–12 — Spring 1999
MW6 CHM-106
Prof. Bumby

Seventh set of workshop problems — Line integrals
Textbook Sections 14.1 – 14.3

1. Compute

$$\int_{\mathcal{P}_n} y\,dx + (3y^3 - x)\,dy + z\,dz$$

for *each* of the paths $\mathcal{P}_n$ from $(0, 0, 1)$ to $(1, 1, 1)$ given by $\mathbf{r}(t) = \langle t, t^n, 1\rangle$ (for $n = 1, 2, , 3, \ldots$).

When combined with the *fundamental theorem for line integrals*, Theorem 2 of Section 14.3, what does this tell you about the vector field

$$\left\langle y, 3y^3 - x, z\right\rangle?$$

2. Find a function $f(x, y)$ such that

$$\nabla f = \left\langle \cos x\left(e^{\sin x}\right)\sin\left(e^y\right), \left(e^{\sin x}\right)e^y\cos\left(e^y\right)\right\rangle,$$

and use the function $f(x, y)$ to find

$$\int_{\mathcal{C}} \cos x\left(e^{\sin x}\right)\sin\left(e^y\right)dx + \left(e^{\sin x}\right)e^y\cos\left(e^y\right)dy,$$

where $\mathcal{C}$ is the portion of the unit circle in the first quadrant from $(0, 1)$ to $(1, 0)$

3.Find

$$\int_{\mathcal{B}} (3x^2 + y)\,dx + (2xy - 3yz)\,dy + (2xz - yz^2 + 4z^2)\,dz,$$

where $\mathcal{B}$ is the path from $(0, 0, 0)$ to $(1, 1, 1)$ given by going along a straight line $\mathcal{B}_1$ from $(0, 0, 0)$ to $(1, 0, 0)$, then along a straight line $\mathcal{B}_2$ from $(1, 0, 0)$ to $(1, 1, 0)$, and finally along a straight line $\mathcal{B}_3$ from $(1, 1, 0)$ to $(1, 1, 1)$. Include parametric equations for each of the segments $\mathcal{B}_1$, $\mathcal{B}_2$, and $\mathcal{B}_3$.

. . . more on other side

4.Let $\mathbf{F}(x, y)$ be the vector field

$$\mathbf{F}(x, y) = \left\langle -\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle.$$

Show that $\mathbf{F} = \nabla\theta$, where $\theta(x, y) = \arctan(y/x)$ is the usual polar angle. Use this to evaluate

$$\int_{\mathcal{R}} \mathbf{F} \cdot d\mathbf{r}$$

where $\mathcal{R}$ is a radial segment from $(\cos\theta, \sin\theta)$ to $(2\cos\theta, 2\sin\theta)$ (i.e. along the straight line through these points whose extension passes through the origin), and

$$\int_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{r}$$

where $\mathcal{S}$ is a semicircle centered at the origin from $(0, -r)$ to $(0, r)$ through the halfplane where $x \geq 0$. Use this to show that

$$\int_{\mathcal{H}} \mathbf{F} \cdot d\mathbf{r} = 0,$$

where $\mathcal{H}$ is the boundary of the “half-washer” consisting of those points (x, y) with

$$x \geq 0 \text{ and } 1 \leq x^2 + y^2 \leq 4.$$

Repeat your calculations for similar curves in the halfplane where $x \leq 0$. You should discover that these integrals are not *always* independent of path. What is wrong with the function giving θ as a function of x nad y to allow a result that appears to contradict the fundamental theorem for line integrals?