Math 251:10–12 — Spring 1999
MW6 CHM-106
Prof. Bumby

Eighth set of workshop problems — Double integrals
Textbook Sections 13.1 – 13.4

1. Decide whether each of the formulas below makes sense as a double integral. For the formulas that you reject, explain your reasoning in one or two sentences; for the legitimate ones, sketch the region of integration in the plane and evaluate the integral.

a) $\int_0^1 \int_0^{3x} (x^3 + 2y + 4)\,dx\,dy$

b) $\int_0^1 \int_0^{3x} (x^3 + 2y + 4)\,dy\,dx$

c) $\int_0^{3x} \int_0^{1} (x^3 + 2y + 4)\,dx\,dy$

d) $\int_1^3 \int_{y-1}^{1-y^2} (x^3 + 2y + 4)\,dx\,dy$

e) $\int_0^1 \int_{y-1}^{1-y^2} (x^3 + 2y + 4)\,dx\,dy$

f) $\int_0^{x^3} \int_{y-1}^{1-y^2} (x^3 + 2y + 4)\,dx\,dy$

2. Consider the iterated integral

$$\int_0^{a \sin\beta} \int_{y \cot\beta}^{\sqrt{a^2-y^2}} \ln(x^2 + y^2)\,dx\,dy,$$

where $a > 0$ and $0 < \beta < \pi/2$.

(a) Sketch the region of integration.

(b) Rewrite the integral with the order of integration reversed.

(c) Rewrite the integral in polar coordinates.

(d) Evaluate the integral written in polar coordinates.

. . . more on other side

3. For each of the following integrals: (1) describe (both in words and with a sketch) the region of integration and evaluate the integral; (2) then rewrite the integral with order of integration reversed and evaluate the integral using this expression; (3) indicate (with a brief statement supporting your claim) which computation was easier.

$$\int_0^1 \int_{x^2}^1 x\sqrt{1-y^2}\,dy\,dx \qquad (a)$$

$$\int_1^2 \int_1^y \frac{y}{\sqrt{x^2+y^2}}\,dx\,dy \qquad (b)$$

$$\int_1^4 \int_1^{\sqrt{x}} \frac{e^{xy^{-2}}}{y^5}\,dy\,dx \qquad (c)$$

Hint for (c): integrate by parts with $u = \dfrac{1}{y^2}$ and $dv = \dfrac{e^{xy^{-2}}}{y^3}\,dy$.