Math 291 Spring 2006: Formulas for Exam #2

$$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos(\theta) \quad \text{and} \quad |\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin(\theta)$$

The area of a parallelogram spanned by $\mathbf{a}$ and $\mathbf{b}$ is $|\mathbf{a}\times\mathbf{b}|$.
The volume of a parallelepiped spanned by $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ is $|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|$.

The tangent plane to $F(x,y,z)=K$ at (a,b,c) is: $\nabla F(a,b,c)\cdot\langle x-a, y-b, z-c\rangle = 0$.

$$\nabla f(x,y,z) = \langle f_x(x,y,z), f_y(x,y,z), f_z(x,y,z)\rangle \text{ and } D_{\mathbf{u}} f(x,y,z) = \nabla f(x,y,z)\cdot\mathbf{u}.$$

Arc length from a to t is given by the function $s(t) = \int_a^t |\mathbf{r}'(u)|\,du$. Thus $s'(t) = |\mathbf{r}'(t)|$.

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} \qquad \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} \qquad \kappa(t) = \frac{d\mathbf{T}}{ds} = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

Suppose that a large object of mass M is located at the origin and an object of mass m is located at the point $\langle x,y,z\rangle$. Then the gravatational force of M on m is $\mathbf{F}(x,y,z) = -\frac{mMG}{\sqrt{x^2+y^2+z^2}}\langle x,y,z\rangle$ where G is the gravatational constant.

Center of mass - 3 dimensions

$$\text{mass} = m = \iiint_E \rho(x,y,z)\,dV \qquad (\bar{x},\bar{y},\bar{z}) = \frac{1}{m}(M_{yz}, M_{xz}, M_{xy})$$

$$M_{yz} = \iiint_E x\,\rho(x,y,z)\,dV, \qquad M_{xz} = \iiint_E y\,\rho(x,y,z)\,dV, \qquad M_{xy} = \iiint_E z\,\rho(x,y,z)\,dV$$

Center of mass - a wire in the plane

$$\text{mass} = m = \int_C \rho(x,y)\,ds \qquad (\bar{x},\bar{y}) = \frac{1}{m}(M_y, M_x)$$

$$M_y = \int_C x\,\rho(x,y)\,ds, \qquad M_x = \int_C y\,\rho(x,y)\,ds$$

Polar: $x = r\cos(\theta)$, $y = r\sin(\theta)$, $dA = r\,drd\theta$

Cylindrical: $x = r\cos(\theta)$, $y = r\sin(\theta)$, $z = z$, $dV = r\,drd\theta dz$

Spherical: $x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, $z = \rho\cos(\phi)$, $dV = \rho^2\sin(\phi)\,d\rho d\theta d\phi$

Change of variables:

$$\iint_R f(x,y)\,dx\,dy = \iint_S f(x(u,v),y(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\,du\,dv$$

$$\iiint_R f(x,y,z)\,dx\,dy\,dz = \iiint_S f(x(u,v,w),y(u,v,w),z(u,v,w))\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right|\,du\,dv\,dw$$

$$ds = |\mathbf{r}'(t)|dt \qquad dx = x'(t)dt \qquad \int_C \mathbf{F}\cdot d\mathbf{r} = \int_C \mathbf{F}\cdot\mathbf{T}\,ds$$

Green's Theorem: $\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\,dA = \iint_D \text{curl}(\mathbf{F})\cdot\mathbf{k}\,dA$