Math 291 – Extra Review Problems for Exam #2

OK! I admit it. These first few problems are just extra workshop problems...but they make good review problems too.

NOTE: Problems 5–7 provide some extra review for sections 16.4 and 16.5.

1. We denote the region bounded by the sphere of radius 4 centered at the origin by S. Let E be the part of S which lies in the first octant. Evaluate $\iiint_E xyz\,dV$.

   (a) using rectangular coordinates.

   (b) using cylidrical coordinates.

   (c) using spherical coordniates.

2. Vector Fields

   (a) Give a formula $\mathbf{F}(x,y) = M(x,y)\mathbf{i} + N(x,y)\mathbf{j}$ for the vector field in the plane that has the property that $\mathbf{F}$ always points towards the origin with magnitude inversely proportional to the square of the distance from (x,y) to the origin. (Obviously, $\mathbf{F}$ is undefined at the origin.)

   (b) Give a formula $\mathbf{F}(x,y) = M(x,y)\mathbf{i} + N(x,y)\mathbf{j}$ for the vector field in the plane that has the property that $\mathbf{F} = \mathbf{0}$ at $(0,0)$ and that at any other point (a,b), $\mathbf{F}$ is tangent to the circle $x^2 + y^2 = a^2 + b^2$ and points in the clockwise direction with magnitude $|\mathbf{F}| = \sqrt{a^2 + b^2}$.

3. Find the work done by the force field

$$\mathbf{F}(x,y) = x\sin(y)\mathbf{i} + y\mathbf{j}$$

   on a particle that moves along the parabola $y = x^2$ from $(-1,1)$ to $(2,4)$.

4. Suppose that $\mathbf{F}(x,y)$ is a (non-zero) *constant* vector field: there is some fixed (non-zero) vector $\mathbf{v}$ we have that $\mathbf{F}(x,y) = \mathbf{v}$ for all (x,y). For what straight line paths C (described parametrically $\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{u}_0 t$, $a \le t \le b$) will $\int_C \mathbf{F} \cdot d\mathbf{r}$ be negative? be zero?

5. Suppose that $\mathbf{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$ is a vector field defined on all of $\mathbb{R}^2$ except at the points $(1,1)$, $(0,1)$, and $(-1,0)$. In fact, $\mathbf{F}$ is so nice that $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$ (everywhere F is defined). Let C_1 be a circle of radius $1/10$ centered at $(1,1)$ and oriented positively. Let C_2 be a circle of radius $1/20$ centered at $(0,1)$ and oriented negatively. Let C_3 be a circle of radius $1/5$ centered at $(-1,0)$ and oriented positively. Furthermore:

$$\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = 1 \qquad \int_{C_2} \mathbf{F} \cdot d\mathbf{r} = 2 \qquad \int_{C_3} \mathbf{F} \cdot d\mathbf{r} = 3$$

   Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is a positively oriented circle centered at the origin of radius 3.

6. Section 16.4: Try problems 1, 7, 12, 21, 22, and 24.

7. Section 16.5: Try problems 16–19, 26, 28, and 32.