WRITING PROOFS

Most of the homework, workshop, and exam problems in this course will ask you to write proofs; your work will be graded carefully for both content and style. We will be talking about how to write proofs throughout the course, but here are some general guidelines.

Content—the logic of the proof

- A proof should contain a clear and rigorous chain of reasoning leading from the hypothesis to the conclusion. You should give arguments supporting all the steps. (More precisely, give arguments for all the steps except for those which are “obvious”. Learning what this means—i.e., learning how much to write down for each argument—is part of learning the art of writing proofs). Everything written down should be relevant to this chain of reasoning: don’t start by writing down a list of things you know, and don’t digress as you go along.

- The most common error that students make is to work backward from the desired conclusion to the given hypotheses. This may be helpful at the preliminary stage of figuring out how to construct a proof, but is never correct for the final version; such work belongs only on scratch paper. You must rewrite the proof, moving from what you know to what you want to establish, to be sure that the logic works in that direction. Another way to say this is that in a proof you should never write down a statement or equality unless you know that it is true—from definitions or previously established results, from the hypotheses which are given, or from some chain of reasoning based on these—or unless you state explicitly that it has not been established (for example, a proof by contradiction might begin “We proceed by contradiction; suppose then that such-and-such is true.”)

- When you are done, read your proof critically. Pretend that it was written by a stranger, and that you did not know what he or she was thinking. Does the proof then convince you absolutely? If not, try again.

Style—the language of the proof

- Write in complete, grammatically proper sentences. Remember that the equal sign is a verb. Avoid dangling modifiers.

- Study proofs in the text, in other books, and from the lectures, to get a feeling for good mathematical style.

- Don’t use the notations ∀ and ∃. Don’t introduce mysterious abbreviations to save writing out words. You are allowed to use the abbreviation “iff” for “if and only if”.

- Read the document Writing Proofs, by Professor James Munkres of MIT, which is posted on the class web page.

Homework and workshop write-ups—when you have time to do it right

- Start the homework early! Then, if you just don’t know how to get started or you get lost in the middle, talk to me, to our workshop instructor Nathaniel Shar, or to other students.

- Don’t turn in scratch work. Once you have decided how to construct a proof, and written out the details once, rewrite the proof neatly to turn in.

- When your work is returned to you, read the comments and be sure you understand their point. If you don’t, come to see me or Nathaniel to talk about them.