Math 321 — Fall 2006
MW5 SEC-220
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Comments on first lecture 09/06/06

Comments on quiz 0

The quiz dealt with the equation

$$\frac{d^2y}{dt^2} + 9y = 0 \tag{0}$$

The standard way to solve a homogeneous linear differential equation with constant coefficients (of any order) is to look for **special solutions** of the form $y = e^{\alpha t}$. Linearity tells you that arbitrary **linear combinations** of these solutions are also solutions to the equation. If you find enough special solutions, coefficients in the linear combinations can be found to match any given **initial solutions**. The existence and uniqueness theorems then say that you have found **all** solutions of the equation. Sometimes you meet an equation that is more complicated, but this approach always gives **some** solutions and usually gives all solutions.

Several students attempted to treat the equation as **separable**. While this is a powerful general method, it only applies to **first order equations**. However, equation (**0**) is **second order**. Because the independent variable t only appears as part of the differential operator d/dt, there is a method for solving this equation that resembles the method used to solve separable equations. The idea is to modify equation (**0**) so that y becomes the independent variable. To do this, we need to introduce a new **dependent** variable. Our choice will be $v = dy/dt$. The key observation is that the **chain rule** says

$$\frac{d^2y}{dt^2} = \frac{dv}{dt} = \frac{dv}{dy} \cdot \frac{dy}{dt} = \frac{dv}{dy} \cdot v.$$

Thus, equation (**0**) is

$$v\frac{dv}{dy} + 9y = 0,$$

which separates as

$$v\,dv + 9y\,dy = 0.$$

The solutions of this equation have $v^2 + 9y^2$ constant. This can be solved for v, and v replaced by dy/dt to obtain another separable equation. If you remember the value of

$$\int \frac{dy}{\sqrt{a^2 - y^2}},$$

you get an expression for t in terms of y involving **two** constants of integration. Inverting this function gives y as a function of t.

Although this is less satisfactory than a solution based on recognizing the equation as linear, it is still a useful general method that is worthy of study.

more ...

Exercise 2.1

Some time in lecture was spent on this exercise which is the simplest example showing that the **vector sum** of the **external forces** on a system is equal to the **total mass** times the acceleration of the **center of mass**. The rest of the motion of the system consists of a **rotation** about the center of mass, but this was not explored.

Section 4

If the spring is vertical, there is the force of gravity to be considered. Equating the **total force** to mass times acceleration gives a differential equation. We will see more examples of this method later: each new force is simply added to the forces that were part of a previous model to produce a modified differential equation. Typically, the equations look similar, but the solutions can be very different. In this case, the equation remains linear, but is no longer homogeneous. Solving such an equation requires that we find a **particular solution**. Then, the difference between the dependent variable and the particular solution satisfies a homogeneous linear equation. In this case, the particular solution is constant, so we wind up with an equation that resembles the original equation, but the variable has been shifted. As long as it is still reasonable to assume that Hooke's law holds with a spring **constant**, a new **rest position** of the spring stretched by the action of gravity has been found. Since we have recovered the original equation, the motion of the system can be described using the same functions appearing in the original solution.

End of Lecture 01