

Math 336, Reading Assignments and Problem Sets, Fall 2008

Week 1

A. Due September 4 (do not hand in): Read the differential equations modeling review at <http://www.math.rutgers.edu/~sontag/336/modeling.pdf> and do the problems therein.

Read sections 4.1 and 6.1 in Edelstein-Keshet.

B. Due September 9:

A. The Beverton-Holt population growth model is

$$\frac{dN}{dt} = \frac{rN}{\alpha + N}, \quad (1)$$

where α has the dimensions of N , namely $[n]$ where n is number of individuals, and r has dimensions of $[n/s]$. Find a dimensionless scalings $N^* = k_1N$ and $t^* = k_2t$, to derive the dimensionless form of the Beverton-Holt model:

$$\frac{dN^*}{dt^*} = \frac{N^*}{1 + N^*}. \quad (2)$$

Determine explicitly what k_1 and k_2 are and give the derivation of equation (2) from equation (1).

B. Find the dimensionless form of the Ricker population model

$$\frac{dN}{dt} = rNe^{-\beta N}.$$

Here r has dimension $[1/s]$ and β has dimension $[1/n]$.

C. Study the derivation of the logistic model on page 119 of Edelstein-Keshet. Now suppose instead that $\kappa(C) = \alpha C/(\beta + C)$, where $\alpha > 0$ and $\beta > 0$. This kind of dependence is expected if the rate of growth as a function of nutrient concentration saturates at high nutrient concentrations. What equation do you then find for N instead of equation (8)?

Edelsteing-Keshet: problems 1b), 2, 3, 4, 6, pages 257-258.

Note about 2 c): Solve the equation explicitly using partial fractions following the method of solving the logistic equation. Make a qualitative sketch of the different possible behaviors of solutions. (You can also plot exact solutions for various initial conditions using computational software, if you like, but this is not necessary.)

Of these problems, hand in: A, C, 2, 4.