Math 336, Problem Set 12, Fall 2008

Read sections 9.5 and 9.6 of Professor Sontag's notes. (You would do well to read the whole chapter to get another perspective on what we have been doing from Edelstein-Keshet.)

Hand in problems 1 and 2 below. For problem 2 (which is problem 1 on page 94 of the Sontag notes), hand in only cases 3, 4, and 6.

1. Read appendices A.1-A.3 to Chapter 9 of Edelstein-Keshet. (We covered A.1 and A.2 in class Tuesday. Professor Butler discussed A.3 the previous Tuesday. If you were not there, the material can be understood pretty easily, and basically you should understand that (119) gives "separated" solutions to a class of heat equations with boundary conditions, which can be used to construct new solutions by linear combination in problems like Example 5.)

(a) Do problem 23 on page 435.

(b) Do problem 26, page 435. To do this use (114) and the analysis of example 5 on page 430, equations (115)-(119). For part (b) of problem 26, use the fact that linear combinations of solutions to equation (106) with the boundary conditions in (115a) are still solutions with the same boundary conditions—you can easily verify this.

(What is the steady state solution to (106) with boundary conditions (115a)? Part of the reason for doing this problem is for you to notice that the solutions you obtain are converging to this steady state solution at rates that depend on the diffusion coefficient D (in the problem this is called $\mathcal{D}$). Is the convergence faster or slower if D is larger and how does this accord with the meaning of D?)

2. Problem 1 on page 94 of the Sontag notes.

3,. Problem 2 on page 95 of the Sontag notes.

4. Problem at the bottom of page 98 of the Sontag notes.

5. Problem at the bottom of page 99 of the Sontag notes.