1. Let $F$ be a field. An affine transformation of $F$ is a function of the form $f(x) = ax + b$ for some nonzero $a \in F$ and some $b \in F$. A dilation is an affine transformation which sends 0 to 0. A translation is an affine transformation of form $t(x) = x + b$.

   a) Show that the set $A$ of affine transformations form a group under compositions. Show that the subset $S$ of dilations is a subgroup, and that the subset $T$ of translations is a subgroup.

   b) Is $S$ a normal subgroup of the group of affine transformations? Is $T$ a normal subgroup of $A$?

   c) When $F$ is a field with a prime $p$ number of elements compute the order of $A$. When $p = 3$ which familiar group is this isomorphic to?

2. Let $G$ be a finite group with subgroups $H, K$. Let $HK = \{hk | h \in H, k \in K\}$.

   a) Show that if $H \cap K$ has only one element, then the size of $HK$ is the product of the orders of $H$ and $K$.

   b) Show that if $K$ is a normal subgroup of $G$ then $HK$ is a subgroup of $G$. Give an example to show that if neither of $H, K$ are normal subgroups, then $HK$ may not be a subgroup.