1. Let $G$ be a group and let $a, b \in G$.
   a) Show that $a$ and $b^{-1}ab$ have the same order.
   b) Show that $a$ and $a^{-1}$ have the same order.
   c) Suppose that every element $g \in G$ satisfies $g = g^{-1}$. Show that $G$ is commutative.
   d) Suppose that every element $g \in G$ satisfies $g^2$ is the identity element. Show that $G$ is commutative.

2. Let $G$ be the distance preserving symmetries of the real line which preserve distance. Find two distinct elements $a$ and $b$ of $G$ of order 2. What is the order of $ab$?

3. Let $G = \langle a \rangle$ be the cyclic group generated by $a$.
   a) If $a$ has infinite order, find the order of each element of $G$.
   b) Redo the previous part in the cases where the order $|a| = 4, 6, 8, \text{ and } 12$.
   c) Suppose that $G$ is generated by an element $a$ of finite order. Find and prove a formula for the order of each element $a^k$ of $G$. 