1. Let $G, H$ be groups and let $\phi : G \to H$ be a group homomorphism. Define the kernel of the homomorphism to be the set $\ker(\phi) = \{ g \in G | \phi(g) = e_H \}$.

   a) Show that $\phi(e_G) = e_H$ and $\phi(g^{-1}) = \phi(g)^{-1}$.

   b) Show that $\ker(\phi)$ is a subgroup of $G$.

2. Let $F$ be a field and let $F^*$ be its multiplicative group of nonzero elements. Let $GL(n, F)$ denote the $n \times n$ matrices with entries in $F$ which have nonzero determinant.

   a) Show that $GL(n, F)$ is a group under matrix multiplication.

   b) Show that taking the determinant of a matrix gives a homomorphism of groups $\det : GL(n, F) \to F^*$.

   c) For each permutation $\sigma$ of $\{1, \ldots, n\}$ let $P_{\sigma}$ denote the matrix obtained by permuting the rows of the identity matrix according to $\sigma$, that is move row $i$ to row $\sigma(i)$. Show that

$$
\begin{pmatrix}
a_1 \\
a_2 \\
\vdots \\
a_n \\
\end{pmatrix} = P_{\sigma} 
\begin{pmatrix}
a_{\sigma(1)} \\
a_{\sigma(2)} \\
\vdots \\
a_{\sigma(n)} \\
\end{pmatrix}
$$

   d) Is the map sending a permutation $\sigma$ to the matrix $P_{\sigma}$ a group homomorphism from $S_n$ to $GL(n, F)$? Provide a proof or explain why it is not a homomorphism.