Later proofs will be easier if we fix notation from the start. That is the main purpose of chapter 1.

Section 1 points out that the word **axiom** in this subject means *one of the defining properties of the system we are studying*. This is what makes the subject **Abstract**. The properties of **groups**, **rings**, and **fields** developed in the course will be proved starting from a few simple axioms As the subject grows, theorems can be found that would have been difficult to prove, or possibly even to state, when the axioms were first met. Yet the systematic development gives us confidence in these results.

What makes the subject **Algebra** is that the main examples are familiar objects like **Integers**, **Real Numbers**, **matrices**, with operations on them like **addition** and **multiplication**. The axioms express properties like **associativity** and **commutativity** that play an important role in our understanding of these operations. The assigned exercise in Section 1 asks you to study the behavior of **subtraction** with respect to possible axioms of algebraic systems.

The other exercises in this section will be discussed in class.

Section 2 introduces the language of **sets** — not really set *theory* as suggested by the title of the section — but a language that will be used to state axioms and describe examples. The operations of **union**, denoted $A \cup B$, and **intersection**, denote $A \cap B$ of sets A and B are described, as is the **empty set**, denoted $\emptyset$. This could be the beginning of an *algebra* of sets, but the only algebraic property that we meet is

$$A \cup B = (A \cap B) \cup (A - B) \cup (B - A),$$

where $A - B$, called the **difference** of A and B denotes the elements of A that are not elements of B.

Another general construction is the **Cartesian product**, denoted $A \times B$ whose elements are **ordered pairs** (a, b).

The basic statements about sets are proved by considering the elements of the sets in the statement. Simple statements are often illustrated with a picture, but the pictures that are usually drawn are too special to be useful except in simple cases.

Exercises 5 thru 11 will be discussed, followed by exercise 20 and 21.

Section 3 introduces the word **mapping** as a synonym for **function**, with the interpretation that it is a rule that assigns elements of one set to elements of another. A mapping f from S to T is denoted

$$f : S \to T.$$

To use the word "mapping" (or "function") we require that the symbol $f(s)$ should have a meaning for **every** $s \in S$, and this meaning should be a **unique** element of T. This allows us to define a **composition** of mappings

$$(g \circ f) : S \to U$$

whenever

$$f : S \to T \text{ and } g : T \to U.$$

The definition is

$$(g \circ f)(s) = g\big(f(s)\big)$$

In other subjects, functions appear to be qualitatively different from the elements to which they apply, and the parentheses reinforce this appearance of difference. In algebra, these parentheses only get in the way. What is **really important** is the convention that the symbol representing the mapping is written to the left of the element to which it applies. Stripped of inessentials, the definition of composition now reads

$$(gf)s = g(fs),$$

which resembles the **associative law**. This leads to the important **Lemma 1.31** which says that composition of mappings obeys the associative law.

Some special properties of mappings that will play an important role are **one-to-one** and **onto**.

Exercises for discussion are 6, 7, 10, 11.