

**Some exercises from Section 2.6.** Exercises 2 thru 5 will be done on the blackboard.

**Homomorphism theorems.** If  $\phi: G \rightarrow G'$  is a homomorphism, the image of  $G$ , i.e.,

$$\begin{aligned} G^* &= \{ g' \in G' : (\exists g \in G) \phi(g) = g' \} \\ &= \{ \phi(g) : g \in G \} \end{aligned}$$

is a subgroup of  $G'$ , so  $\phi$  splits into a map *onto*  $G^*$  followed by the inclusion of  $G^*$  in  $G'$ . Inclusions of subgroups seem almost too trivial to mention, so the mapping onto  $G^*$  gets most of the attention.

The kernel  $K$  of this mapping is the same as the kernel of  $\phi$  since the identity of the subgroup  $G^*$  is the identity element of  $G'$ . The factor group construction leads to another group  $G/K$  with a homomorphism from  $G$  to  $G/K$  having  $K$  as kernel. The **First homomorphism theorem** says that  $G/K$  is isomorphic to  $G^*$  (and hence acts like a subgroup of  $G'$ ) and this isomorphism is consistent with the homomorphisms we have from  $G$  to each of these groups.

The next result mentioned in this section is the **Correspondence theorem** that characterizes the sets

$$H = \{ a \in G : \phi(a) \in H' \},$$

where  $H'$  is a subgroup of  $G'$ , as the subgroups of  $G$  that contain  $K$ . Since  $K$  is a normal subgroup of  $H$ , we can form  $H/K$ . The theorem also relates  $H/K$  to  $H'$ . If  $H'$  is a normal subgroup of  $G'$ ,  $H$  will also be a normal subgroup of  $G$ .

The **Second Homomorphism Theorem** starts with an arbitrary subgroup  $H$  of  $G$  and considers its image in  $G/N$  for some  $N \triangleleft G$ . The image of  $H$  is  $H/H \cap N$ , and the correspondence theorem shows that this is isomorphic to the quotient of a group containing  $N$ , which turns out to be  $HN$  by  $N$ . This gives

$$\frac{H}{H \cap N} \cong \frac{HN}{N}. \quad (II)$$

Finally, there is a **Third Isomorphism Theorem** that looks at the factor group of  $G'$  by a normal subgroup  $N'$  and relates it to  $G/N$  where

$$N = \{ a \in G : \phi(a) \in N' \}.$$

Proofs of these theorems, as well as exercises 2 and 6, will be done at the blackboard. This means that exercise 5 should be removed from the list of homework problems.

Section 2.8, along with its homework, will also be skipped at this time.