Some exercises from Section 2.6. Exercises 2 thru 5 will be done on the blackboard.

Homomorphism theorems. If $\phi: G \to G'$ is a homomorphism, the image of G, i.e.,

$$G^* = \{ g' \in G' : (\exists g \in G)\phi(g) = g' \} \\ = \{ \phi(g) : g \in G \}$$

is a subgroup of G', so ϕ splits into a map *onto* G^* followed by the inclusion of G^* in G'. Inclusions of subgroups seem almost too trivial to mention, so the mapping onto G^* gets most of the attention.

The kernel K of this mapping is the same as the kernel of ϕ since the identity of the subgroup G^* is the identity element of G'. The factor group construction leads to another group G/K with a homomorphism from G to G/K having K as kernel. The **First homomorphism theorem** says that G/K is isomorphic to G^* (and hence acts like a subgroup of G') and this isomorphism is consistent with the homomorphisms we have from G to each of these groups.

The next result mentioned in this section is the **Correspondence theorem** that characterizes the sets

$$H = \{ a \in G : \phi(a) \in H' \},$$

where H' is a subgroup of G', as the subgroups of G that contain K. Since K is a normal subgroup of H, we can form K/H. The theorem also relates H/K to H'. If H' is a normal subgroup of G', H will also be a normal subgroup of G.

The **Second Homomorphism Theorem** starts with an arbitrary subgroup H of G and considers its image in G/N for some $N \triangleleft G$. The image of H is $H/H \cap N$, and the correspondence theorem shows that this is isomorphic to the quotient of a group containing N, which turns out to be HN by N. This gives

$$\frac{H}{H \cap N} \simeq \frac{HN}{N}. \qquad (II)$$

Finally, there is a **Third Isomorphism Theorem** that looks at the factor group of G' by a normal subgroup N' and relates it to G/N where

$$N = \{ a \in G : \phi(a) \in N' \}.$$

Proofs of these theorems, as well as exercises 2 and 6, will be done at the blackboard. This means that exercise 5 should be removed from the list of homework problems.

Section 2.8, along with its homework, will also be skipped at this time.