Math 351:03 — Fall 1999
MW4 SEC-217
Prof. Bumby

Workshop 6, Last look at groups: the Symmetric group.

9*. This problem completes the description the action of S_4 on the set of *pairs* of distinct elements of $S = \{0, 1, 2, 3\}$.

(a) Suppose there is an $a \in S_4$ such that $b' = aba^{-1}$. How are the elements of $A(T)$ determined by b and b' related?

(b) Since elements of S_4 are determined up to inner automorphism by their cycle structure, the result of (a) allows information about the behavior of the single 2-cycle g to determine the behavior of all 2-cycles, and similarly for the 3-cycle h and the 4-cycle k. How many elements of S_4 are not covered by these examples? What are the cycle structures of these elements?

(c) Determine the behavior of the missing elements without examining the action of a typical element on all elements of T.

13. Do exercise 3.2#17. This asks you to show that S_n can be generated by just two elements: the transposition (0 1) and the cycle that takes each element to the next modulo n. The main ingredient is the proof of Theorem 3.2.5 using the "bubble sort".

14. Show that every product of two *involutions* in S_n is of the form g^2 for some $g \in S_n$. (There are two cases depending on whether the cycles are disjoint or not.) Use this to show that the alternating group A_n is generated by all the squares in S_n. To show that the words, "generated by" are required in the last sentence, give an example of an element in A_6 that is not the square of any element of S_6.

End workshop 6