

Math 351:03 — Fall 1999

MW4 SEC-217

Prof. Bumby

Workshop 11, Ideals and isomorphism theorems.

23. Let R be a ring (the integers will be a good example), and let F be the field of fractions of R . Suppose $p(x)$ is an irreducible monic polynomial with coefficients in R . We want to find the ideals in the ring $R[x]/(p(x))$.

The correspondence theorem says that the ideals of this ring can be identified with ideals of $R[x]$ that contain $(p(x))$. Since $p(x)$ is irreducible, each ideal that properly contains $(p(x))$ also contains an element of R .

Instead of trying to describe all ideals at the same time, we try to find the maximal ideals. Certainly, the elements of R that lie in an ideal of $R[x]$ form an ideal of R , so we look at ideal that contain some fixed maximal ideal M of R . An ideal of $R[x]$ containing M contains all multiples of elements of M with elements of $R[x]$. Another description of this set is the set of all polynomials, all of whose coefficients lie in M . Check that this is an ideal.

Again, by the correspondence theorem, the ideals of $R[x]$ containing $M \cdot R[x]$ are the ideals of the factor ring. This factor ring can be shown to be $(R/M)[x]$.

The maximal ideal here are found by factoring $p(x)$ into irreducibles over R/M .