

Some classical applications of algebra

Number theory:

1. In any fraction of the form $1/p$ with p a prime, the length of the periodic part of the fraction is a divisor of $p - 1$. [Fermat] Uses section 7.5.
2. An odd prime number is a sum of two squares if and only if its remainder on division by 4 is 1. [Fermat] Uses chapter 7 and section 9.1.

Classical geometrical problems:

3. In general, it is impossible to trisect a given angle using ruler and compass alone. Chapter 15.
4. A regular polygon with p sides, where p is prime, can be constructed by ruler and compass if and only if p has the form $2^{2^n} + 1$. [Gauss] Uses the ideas of Chapter 15 and Chapter 7.

Solutions to equations:

5. While explicit formulas can be given for the solution of equations of degree 2, 3, or 4, in terms of radicals (n -th roots), no such formula can be given for equations of degree 5. [Abel-Ruffini-Galois] Section 11.3.

Chapters 12-16 of the book include other applications, some very modern—the one in Chapter 12 is essential to internet security (secure log-in and so forth).