354-04: Quiz #5, 4/4/06 *SOLUTIONS*

1. Start to solve the following LPP by the primal-dual method. Carry out one pivot, circle the next pivot, and stop. State explicitly for which values of the parameter, if any, your tableaux are optimal (i.e. feasible and dual feasible).

Maximize $x + 5y$, subject to

$$\begin{aligned} x - y &\geq 1 \\ 3x + 2y &\leq 20 \\ x \geq 0,\ y &\geq 0 \end{aligned}$$

SOLUTION: Convert to standard form: the first constraint is $-x + y \leq -1$. *Initial tableau:*

	x	y	u	v	
u	-1	1	1	0	$-1+\mu$
v	3	2	0	1	$20+\mu$
	$-1+\mu$	$-5+\mu$	0	0	

This is feasible and dual-feasible for all $\mu \geq 5$. *As* μ *decreases past the threshold* $\mu = 5$, *the* $-5+\mu$ *becomes negative, so* y *must enter the basis. The departing variable is determined by the values of* θ*-ratios*

$$\frac{-1+\mu}{1} \text{ and } \frac{20+\mu}{2}$$

for μ *just less than* 5. *These values are about* 4 *and* 25/2, *respectively. The first one is smaller; therefore* u *is the departing variable, and we pivot on the* 1 *in the* y*-column. Next tableau:*

	x	y	u	v	
y	-1	1	1	0	$-1+\mu$
v	5	0	-2	1	$22-\mu$
	$-6+2\mu$	0	$5-\mu$	0	

As μ *decreases from* 5 *to* 0,

$$\begin{aligned} -1+\mu &\text{ becomes negative at } \mu = 1, \\ 22-\mu &\text{ stays nonnegative,} \\ -6+2\mu &\text{ becomes negative at } \mu = 3, \\ 5-\mu &\text{ stays nonnegative.} \end{aligned}$$

So $\mu = 3$ *is the next threshold. The above tableau is feasible and dual-feasible for* $3 \leq \mu \leq 5$. *For* μ *slightly less than* 3, *the first entry of the objective row is negative, so we use simplex method, let* x *enter the basis. No* θ*-ratios are needed, since the only possible pivot (positive!) is the* 5. *Circle the* 5.