Some review problems, exam #2

Disclaimer: These are typical problems, but are not meant to be an exhaustive list. Your homework assignments, together with these problems, give a reliable idea of the kinds of problems that the exam will largely consist of.

0. Be able to solve LPP's by the primal-dual method, pure integer programming problems by using Gomory cutting planes, and pure or mixed integer programming problems by using the branch and bound technique (Dakin's algorithm).

1. Suppose that a (P) is a LPP with an optimal solution. Then we know that the dual problem (D) does as well. If we add a constraint to (P) to make a new problem (P'), then which of the following are possible in principle about the new dual problem (D')?

a) (D') might be unbounded

b) (D') might have an optimal solution, with strictly smaller objective function value than the optimal solution of (D)

c) (D') might have an optimal solution, with exactly the same objective function value as the optimal solution of (D)

d) (D') might be infeasible.

2. Let (P) be the following LPP:
Maximize $4x_1 + 3x_2 + 4x_3$ subject to $4x_1 + 2x_2 + x_3 \le 150$, $x_1 + 2x_2 + 4x_3 \le 20$, all $x_i \ge 0$.

(a) Let (D) be the dual problem. Formulate (D) and solve it geometrically.

(b) From the picture of the dual solution and without further calculations, determine as many primal variables (including slack variables) as you can which are 0 at the optimal solution of the primal. (Complementary slackness!)

(c) Which dual variables are basic at the optimal solution of (D)? Which primal variables are basic at the optimal solution of (P)?

3. A certain integer programming problem has decision variables x and y. As usual, z is the objective function. At a certain point in solving the problem by Dakin's algorithm (branch and bound), suppose that the tree has exactly 4 leaves (A, B, C and D), and that the LPPs that they represent (ignoring integer constraints, as usual) have the following optimal solutions:

(A) $x = 1, y = 6.5, z = 243$. (B) $x = 2, y = 5, z = 230$. (C) $x = 3.2, y = 3, z = 211$. (D) Infeasible

(a) Suppose that in the original problem, x and y are both required to be integers. How should the nodes be marked? What (if anything) should be done next? Is there any choice, and if so, what is the preferred choice?

(b) Same questions, but assume that the original problem was different: only x has been required to be an integer.

(c) Same questions, but assume that only y has been required to be an integer.

4. Suppose that we reach the following tableau in the course of solving a pure integer programming problem using Gomory cutting planes. The original problem had all integer coefficients in the constraints and objective function. The original problem was in standard form.

	1	−1.2	0	0	0.8	3.3
	0	2.4	0	1	−1.4	4.2
	0	−3.6	1	0	3.2	12.8
	0	18	0	0	8	33

(a) Explain why we're not done yet.

(b) Continue the solution to find the next tableau (except the objective row).

(c) What if anything can you say about the objective row of the next tableau, without doing any calculations?

(d) Is a further cutting plane needed? If so, what is it, and if not, why not?

(e) How many decision variables were there in the original problem?

5. A certain LPP in standard form, with decision variables x_1, x_2, x_3 and three constraints, is solved by introducing slack variables u_1, u_2, u_3 for the respective constraints, and an artificial variable a_1 for the first constraint. (The first constraint was $\ge$, the other two were $\le$, and the right sides were all positive.) The 2-phase method is used and the artificial variable goes along for the ride in Phase 2. The final tableau for Phase 2 is

	x_1	x_2	x_3	u_1	u_2	u_3	a_1	
x_3	1	0	1	−5	−1	0	5	6
u_3	2	0	0	1	1	1	−1	8
x_2	−3	1	0	−0.25	0.125	0	0.25	1
	2	0	0	20	4	0	−10	−24

(a) What is the matrix B corresponding to this tableau? (Enough information is given about the original tableau for you to answer this!)

(b) The objective function was $z = -6x_1 - 4x_3$ $(= -6x_1 + 0x_2 - 4x_3)$. If one of the coefficients is changed, what will be the range of its possible values so that optimality still occurs at this BFS? (This is 3 separate questions, depending on which coefficient is changed.)

(c) The original right sides were 2, 4 and 6. If the original problem is modified by changing the 2 to 5, will optimality still occur with the same variables being basic? If so, what will the new optimal values of the basic variables be? If not, what method will quickly give the new optimal solution?

(d) Same as (c), assuming that 2 is changed to −1.

(e) For the dual problem, what is the optimal value for the second decision variable? the first slack variable?

6. You are solving a LPP by the primal-dual method. You have frozen μ at $\mu = 1.25$ and pivoted to the following tableau, which gives an optimal solution for that value of μ:

	x_1	x_2	x_3	x_4	u_1	u_2	
x_3	5	−5	1	0	10	−10	$32 - 8\mu$
x_4	−2	−8	0	1	20	−10	$-3 + 3\mu$
	$5 - 4\mu$	$-2 + 5\mu$	0	0	$-2 + 3\mu$	$-6 + 12\mu$	

Questions: What is the next threshold value of μ, and what is the next pivot?