

Example 4: Maximize $3x_1 - 4x_2 + x_3$ subject to

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &\leq -2 \\ 2x_1 - x_2 &\geq 3 \\ x_2 + 5x_3 &\leq 10 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

	x_1	x_2	x_3	u_1	u_2	u_3	a_1	a_2	
a_1	-1	-2	-3	-1	0	0	1	0	2
a_2	2	-1	0	0	-1	0	0	1	3
u_3	0	1	5	0	0	1	0	0	10
	-1	3	3	1	1	0	0	0	-1

Phase I begins.
 x_1 enters, a_2 departs.

	x_1	x_2	x_3	u_1	u_2	u_3	a_1	a_2	
a_1	0	-5/2	-3	-1	-1/2	0	1	1/2	7/2
x_1	1	-1/2	0	0	-1/2	0	0	1/2	3/2
u_3	0	1	5	0	0	1	0	0	10
	0	5/2	3	1	1/2	0	0	1	-7/2

Phase I ends with nonzero objective function. Original LPP is infeasible

Example 5

Maximize $3x_1 - 4x_2 + x_3$ subject to

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &\leq -2 \\ 2x_1 - x_2 &\geq 3 \\ x_2 + 5x_3 &\leq 10 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

	x_1	x_2	x_3	u_1	u_2	u_3	a_1	a_2	
a_1	-1	-2	3	-1	0	0	1	0	2
a_2	2	-1	0	0	-1	0	0	1	3
u_3	0	1	5	0	0	1	0	0	10
	-1	3	-3	1	1	0	0	0	-1

Phase I begins: x_3 enters, a_1 departs.

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	x_1	x_2	x_3	u_1	u_2	u_3	a_1	a_2	
x_3	-1/3	-2/3	1	-1/3	0	0	1/3	0	2/3
a_2	2	-1	0	0	-1	0	0	1	3
u_3	5/3	13/3	0	5/3	0	1	-5/3	0	20/3
	-2	1	0	0	1	0	1	0	-3

Phase I continues: x_1 enters, a_2 departs.

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	x_1	x_2	x_3	u_1	u_2	u_3			
x_3	0	-5/6	1	-1/3	-1/6	0			7/6
x_1	1	-1/2	0	0	-1/2	0			3/2
u_3	0	31/6	0	5/3	5/6	1			25/6
	0	5/3	0	-1/3	-1/6	0			16/3

Phase II begins. u_1 enters, u_3 departs.

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	x_1	x_2	x_3	u_1	u_2	u_3	a_1	a_2	
x_3	0	-5/6	1	-1/3	-1/6	0	1/3	1/6	7/6
x_1	1	-1/2	0	0	-1/2	0	0	1/2	3/2
u_3	0	31/6	0	5/3	5/6	1	-5/3	5/6	25/6
	0	0	0	0	0	0	1	1	0

Phase I ends. Artificial variables have been driven out.

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	x_1	x_2	x_3	u_1	u_2	u_3			
x_3	0	1/5	1	0	0	1/5			2
x_1	1	-1/2	0	0	-1/2	0			3/2
u_1	0	31/10	0	1	1/2	3/5			5/2
	0	27/10	0	0	0	1/5			13/2

Phase II ends. Optimal solution:
 $x_1 = 3/2$, $x_2 = 0$, $x_3 = 2$, $z = 13/2$.