In the following questions, \( A \) is some \( m \times n \) real matrix and \( b \) is some \( m \times 1 \) real column vector; \( Ax = b \) is thus some unspecified system of \( m \) linear equations in \( n \) unknowns.

In each question, a sentence is begun and given several possible completions. Mark each completion T( rue) or F (alse) according as it makes the sentence true or false.

1. The system \( Ax = b \) necessarily has at least one solution provided that
   \[ T \quad (a) \quad \text{rank}(A) = m. \]
   \[ F \quad (b) \quad \text{rank}(A) = n. \]
   \[ F \quad (c) \quad \text{The columns of } A \text{ form a linearly independent set.} \]
   \[ T \quad (d) \quad \text{The rows of } A \text{ form a linearly independent set.} \]

2. The columns of \( A \) form a linearly independent set if and only if
   \[ T \quad (a) \quad \text{The null space of } A \text{ equals } \{0\}. \]
   \[ F \quad (b) \quad \text{The column space of } A \text{ has dimension equal to } \text{rank}(A). \]

3. The number of nonpivot variables in the system \( Ax = b \) is necessarily
   \[ F \quad (a) \quad \text{rank}(A) - \text{rank}(b). \]
   \[ T \quad (b) \quad n - \text{rank}(A). \]
   \[ F \quad (c) \quad \text{The largest number of columns of } A \text{ forming a linearly independent set.} \]
   \[ F \quad (d) \quad \text{rank}(A) - m. \]

4. Assuming that the system \( Ax = b \) has a solution,
   \[ T \quad (a) \quad \text{Setting all the nonpivot variables equal to 0 determines a unique solution of the system.} \]
   \[ F \quad (b) \quad \text{There must be a solution of the system in which all the pivot variables equal 0.} \]