1. (a) Explain why \[
\begin{bmatrix}
0 \\
4 \\
0 \\
-1
\end{bmatrix}
\] is or is not a basic solution of the system

\[
\begin{align*}
2x_1 + x_2 - 3x_3 - 3x_4 &= 7 \\
4x_1 - 3x_2 - 5x_3 + 9x_4 &= -21
\end{align*}
\].

(b) For the same system of equations, find the basic solution (if any) in which \(x_1\) and \(x_3\) are the basic variables.

SOLUTION: The given vector is a solution – it satisfies the equations. There are \(m = 2\) equations, so a basic solution must have 2 basic variables. Since nonbasic variables are 0, the only choice is for \(x_2\) and \(x_4\) to be basic. Now, the columns \[
\begin{bmatrix}
1 \\
-3
\end{bmatrix}, \begin{bmatrix}
-3 \\
9
\end{bmatrix}
\] are not linearly independent, and so the solution is not basic.

(b) Since the columns of coefficients corresponding to \(x_1\) and \(x_3\) are linearly independent, there is such a basic solution, and in it, \(x_2 = x_4 = 0\). Substituting these 0’s in the original equations and solving for \(x_1\) and \(x_3\) gives \(x = \begin{bmatrix}
-49 \\
0 \\
-35 \\
0
\end{bmatrix}\).
1. (a) Explain why \[
\begin{bmatrix}
0 \\
7 \\
2 \\
0
\end{bmatrix}
\]
is or is not a basic solution of the system
\[
\begin{align*}
5x_1 + x_2 - 2x_3 - 4x_4 &= 3 \\
3x_1 - 3x_2 + 6x_3 - 2x_4 &= -9
\end{align*}
\]

(b) For the same system of equations, find the basic solution (if any) in which \(x_1\) and \(x_4\) are the basic variables.

SOLUTION: The given vector is a solution – it satisfies the equations. There are \(m = 2\) equations, so a basic solution must have 2 basic variables. Since nonbasic variables are 0, the only choice is for \(x_2\) and \(x_3\) to be basic. Now, the columns \[
\begin{bmatrix}
1 \\
-3
\end{bmatrix}, \begin{bmatrix}
-2 \\
6
\end{bmatrix}
\]
are not linearly independent, and so the solution is not basic.

(b) Since the columns of coefficients corresponding to \(x_1\) and \(x_4\) are linearly independent, there is such a basic solution, and in it, \(x_2 = x_3 = 0\). Substituting these 0’s in the original equations and solving for \(x_1\) and \(x_4\) gives
\[
\begin{bmatrix}
x_1 \\
x_4
\end{bmatrix} = \begin{bmatrix}
-21 \\
0 \\
0 \\
-27
\end{bmatrix}.
\]