Computational Part: Please submit the program you wrote!

(1) Approximate \( \int_0^2 x^2 \sin(-x) \, dx \approx -2.4694834 \) by the following quadrature rules to \( 10^{-6} \) accuracy and also find the size of \( h \) required for each rule.

(a) Composite left point rule.
(b) Composite right point rule.
(c) Composite midpoint rule.
(d) Composite trapezoidal rule.
(e) Composite simpson’s rule.

Theoretical Part:

(1) Consider the numerical quadrature rule to approximate \( \int_0^1 f(x) \, dx \) given by

\[
\int_0^1 f(x) \, dx \approx w_1 f(0) + w_2 f(x_1).
\]

Find the maximum possible degree of precision you can attain by appropriate choices of \( x_1, w_1 \) and \( w_2 \). By such choices of \( x_1, w_1 \) and \( w_2 \), approximate \( \int_0^1 x^3 \, dx \) and compare with the exact value.

(2) (Optional!!) Show that if \( n \) is even, we have

\[
\sum_{i=0}^{n} w_i \left( x_i - \frac{a + b}{2} \right)^{n+1} = 0,
\]

where \( x_i = a + ih \) with \( i = 0, \cdots, n \) and

\[
w_i = \int_a^b L_k(x) \, dx,
\]

where \( L_k \) is the k-th basis of Lagrange interpolating polynomial.
(3) Determine constants $a, b, c$ and $d$ that will produce a quadrature formula
\[ \int_{-1}^{1} f(x) \, dx = af(-1) + bf(1) + cf'(-1) + df'(1). \]
that has degree of precision 3.

The following problems is from §4.3 in your textbook.
(4) Exercise # 1. (a. and b.)
(5) Exercise # 13.
(6) Exercise # 15.
(7) Exercise # 20.
(8) Exercise # 24.