Math 373 — Spring 2000

Professor Barbara Osofsky

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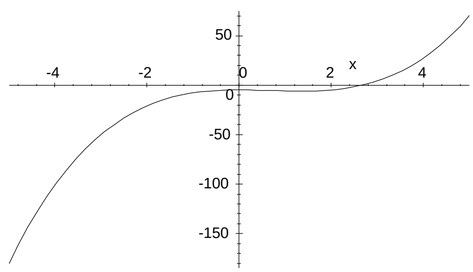
Answers to Homework 7 (lecture 7 – Due 2/15/00)

The odd numbered exercises have answers in the book, so you can check your work. Only a few will be answered here. Even numbered ones are answered here.

Exercise 1 (Page 100: 1a) *Find approximations to withinn* 10^{-4} *to all the real zeros of the polynomial* $P(x) = x^3 - 2x^2 - 5$.

First let us graph the function to see how many real roots to find and where we should start.



It looks like there is only one real root, which is confirmed by noting that $P'(x) = 3x^2 - 4x$ has zeros at $x = 0$ and $x = 4/3$, and $P(0) = 5 < 0$, $P(4/3) = -\frac{167}{27} < 0$. Let us start our iteration with $p_0 = 3$.

$p_1 = p_0 - \frac{P(p_0)}{P'(p_0)}$ and we use Horner's method to evaluate the polynomial values $P(3)$ and $P'(3)$, to show that we understand how it works. Initially I will copy the bottom line of one synthetic division over to a new application of Horner's Method, and in other problems I will simply work under the computations for computing $P(x_0)$ to get $P'(x_0)$ as the algorithm actually does it. You cannot tell sequencing from this static printout.

					⋮				
3	1	−2	0	−5	⋮	3	1	1	3
		3	3	9	⋮			3	12
	1	1	3	4	⋮		1	4	15

so $p_1 = 3 - \frac{4}{15} = 2.7333333$.

					⋮				
2.7333333	1	−2	0	−5	⋮	2.7333333	1	.7333333	2.0044443
		2.7333333	2.0044443	5.4788144	⋮			2.7333333	9.4755553
	1	.7333333	2.0044443	.4788144	⋮		1	3.4666666	11.48

so $p_2 = 2.7333333 - \frac{.4788144}{11.48} = 2.6916247$.

					⋮				
2.6916247	1	−2	0	−5	⋮	2.6916247	1	.6916247	1.8615941
		2.6916247	1.8615941	5.0107127	⋮			2.6916247	9.1064377
	1	.6916247	1.8615941	.0107127	⋮		1	3.3832494	10.968032

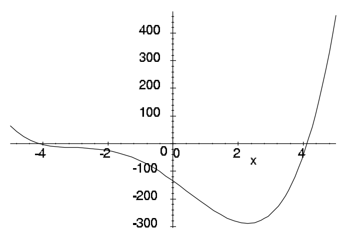
so $p_3 = 2.6916247 - \frac{.0107127}{10.968032} = 2.690648$.

					⋮				
2.690648	1	−2	0	−5	⋮	2.690648	1	.690648	1.8582907
		2.690648	1.8582907	5.0000062	⋮			2.690648	9.1031332
	1	.690648	1.8582907	.0000062	⋮		1	3.3832494	10.961424

so $p_3 = 2.690648 - \frac{.0000062}{10.961424} = 2.6906474$ and we are well within 10^{-4} of our previous iterate, so return the approximate root 2.6906474.

Exercise 2 (Page 100: 2a, 4a) *Approximate all zeros of the polynomial* $P(x) = x^4 + 5x^3 - 9x^2 - 85x - 136$. *First use Newton and reduction of order, and then Müller's Method.*

Let us first graph to get a rough idea where to start.



Aha! There are two real roots, near ± 4. Let us look for the one near $+4$, starting at $p_0 = 4$ and using Horner's Method in the form of synthetic division.

4	1	5	−9	−85	−136
		4	36	108	92
	1	9	27	23 ‖	−44 ‖
		4	52	316	
	1	13	79 ‖	339 ‖	

so $p_1 = p_0 - \frac{P(p_0)}{P'(p_0)} = 4 - \frac{(-44)}{339} = 4.1297935$.

4.1297935	1	5	−9	−85	−136
		4.1297935	37.704162	118.54226	138.52261
	1	9.1297935	28.704162	33.54226 ‖	2.52261 ‖
		4.1297935	54.759356	344.68709	
	1	13.259587	83.463518 ‖	378.22935 ‖	

so $p_2 = p_1 - \frac{P(p_1)}{P'(p_1)} = 4.1297935 - \frac{2.52261}{378.22935} = 4.123124$.

4.123124	1	5	−9	−85	−136
		4.123124	37.615772	117.98638	136.00694
	1	9.123124	28.615772	32.98638	.00694
		4.123124	54.615923	343.1746	
	1	13.246248	83.231695	376.16098	

so $p_3 = p_2 - \frac{P(p_2)}{P'(p_2)} = 4.123124 - \frac{.00694}{376.16098} = 4.1231056.$

4.1231056	1	5	−9	−85	−136
		4.1231056	37.615528	117.98484	135.99998
	1	9.1231056	28.615528	32.98484	$2. \times 10^{-5}$
		4.1231056	54.615527	343.17043	
	1	13.246211	83.231055	376.15527	

so $p_4 = p_3 - \frac{P(p_3)}{P'(p_3)} = 4.1231056 - \frac{2. \times 10^{-5}}{376.15527} = 4.1231055$. We are clearly at the limits of the precision of the arithmetic used so return one approximate root 4.1231055 and the quotient polynomial $Q(x) = x^3 + 9.1231056x^2 + 28.615528x + 32.98484$ which should have as roots the other three roots of $P(x)$. We should have probably used higher precision, as there have been round-off errors affecting the 5^{th} place after the decimal point. We now work with this $Q(x)$ and a starting $p_0 = -4$.

−4	1	9.1231056	28.615528	32.98484
		−4	−20.492422	−32.492424
	1	5.1231056	8.123106	.492416
		−4	−4.4924224	
	1	1.1231056	3.6306836	

so $p_1 = p_0 - \frac{Q(p_0)}{Q'(p_0)} = -4 - \frac{.492416}{3.6306836} = -4.1356263.$

−4.1356263	1	9.1231056	28.615528	32.98484
		−4.1356263	−20.626351	−33.040251
	1	4.9874793	7.989177	−.055411
		−4.1356263	−3.5229457	
	1	.851853	4.4662313	

so $p_2 = p_1 - \frac{Q(p_1)}{Q'(p_1)} = -4.1356263 - \frac{-.055411}{4.4662313} = -4.1232196.$

−4.1232196	1	9.1231056	28.615528	32.98484
		−4.1232196	−20.615628	−32.985344
	1	4.999886	7.9999	−.000504
		−4.1232196	−3.6146881	
	1	.8766664	4.3852119	

so $p_3 = p_2 - \frac{Q(p_2)}{Q'(p_2)} = -4.1232196 - \frac{-.000504}{4.3852119} = -4.1231047.$

−4.1231047	1	9.1231056	28.615528	32.98484
		−4.1231047	−20.615527	−32.984842
	1	5.0000009	8.000001	-2.0×10^{-6}
		−4.1231047	−3.6155348	
	1	.8768962	4.3844662	

so $p_4 = p_3 - \frac{Q(p_3)}{Q'(p_3)} = -4.1231047 - \frac{-2.0 \times 10^{-6}}{4.3844662} = -4.1231042.$

We are clearly at the limits of our precision (as well as less than 10^{-5} between successive iterates, so return another approximate root of -4.1231042 and a quotient polynomial having the remaining roots $R(x) = x^2 + 5x + 8$ with roots $r_1 = \frac{-5-\sqrt{5^2-4*8}}{2} = -2.5 - 1.3228757i$ and $r_2 = \frac{8}{r_1} = -2.5 + 1.3228757i$.

Since we know the roots, I will just do a sample iteration of Müller's method and stop showing the synthetic division (although on an exam if you are asked to use Horner's method to evaluate polynomials you must show it or an equivalent). Let us start with three points, say $x_0 = -4$, $x_1 = 0$, $x_2 = 4$. We now follow algorithm 2.8 in the book, page 98.

STEP 1. Set $h_1 = x_1 - x_0 = 4$, $h_2 = x_2 - x_1 = 4$, $\delta_1 = \frac{P(0)-P(-4)}{4} = -33$, $\delta_2 = \frac{P(4)-P(0)}{4} = 23$, $d = \frac{\delta_2-\delta_1}{h_1+h_2} = \frac{56}{8} = 7$.

STEP 2. We proceed to go through the main loop of the algorithm.

STEP 3. $b = \delta_2 + h_2 d = 23 + 4*7 = 51$. $D = \sqrt{(b^2 - 4P(x_2)\,d)} = \sqrt{(51)^2 - 4 * P(4) * 7} = 61.911227$.

STEP 4. Since b and D are both positive, we set $E = b + D = 51 + 61.911227 = 112.91123$.

STEP 5. Set $h = -2\frac{P(x_2)}{E} = -2\frac{P(4)}{112.91123} = .77937332$, $p = x_2 + h = 4.77937332$.

STEP 6. We are not done, so we proceed to the next step.

STEP 7. Set $x_0 = 0$, $x_1 = 4$, $x_2 = 4.77937332$, $h_1 = 4$, $h_2 = .77937332$, $\delta_1 = \frac{P(4)-P(0)}{4} = 23$, $\delta_2 = \frac{P(4.77937332)-P(4)}{.77937332} = 466.79717$, $d = \frac{(466.79717-23)}{4+.77937332} = 92.85677$ and go to the start of the main loop.

STEP 3. $b = \delta_2 + h_2 d = 466.79717 + (.77937332)(92.85677) = 539.16726$ and

$$D = \sqrt{(539.16726)^2 - 4 * P(4.77937332) * (92.85677)} = 414.62696.$$

STEP 4. Since b and D are both positive, set $E = 539.16726 + 414.62696 = 953.79422$.

STEP 5. Set $h = -2\frac{P(4.77937332)}{953.79422} = -.67060431$, $p = 4.77937332 + (-.6706043) = 4.108769$.

STEP 6. We are not yet done, but at least from outside the program we know we have three points close to the root. I will not continue iterating, nor will I try to find initial values which will have Müller's Method converge to the other roots.