

(a) Data to interpolate

$$f(1) = 2$$

$$f(3) = -4$$

$$f(6) = 5$$

| i | x_i | $f[x_i]$ | $f[x_i, x_{i+1}]$ | $f[x_i, x_{i+1}, x_{i+2}]$ | $f[x_i, x_{i+1}, x_{i+2}, x_{i+3}]$ |
|---|-------|----------|-------------------|----------------------------|-------------------------------------|
| 0 | 1 | 2 | | | |
| | | | -3 | | |
| 1 | 3 | -4 | | 1.2 | |
| | | | 3 | | -0.44 |
| 2 | 6 | 5 | | -1 | |
| | | | 0 | | |
| 3 | 6 | 5 | | | |

(b) $P(x) = 2 + (x-1) * (-3 + (x-3) * (1.2))$

$$\begin{aligned} P(4) &= 2 + (4-1) * (-3 + (4-3) * 1.2) \\ &= 2 + 3 * (-3 + 1.2) \\ &= 2 + 3 * (-1.8) = 2 - 5.4 = -3.4 \end{aligned}$$

(c) Then add $f'(6) = 0$

$$P_{\text{new}}(x) = 2 + (x-1) * (-3 + (x-3) * (1.2 + (x-6) * (-0.44)))$$