

Tentative Syllabus for

Introduction to the Foundations of Quantum Mechanics

Instructor: One of S. Goldstein, M. Kiessling, R. Tumulka

26 lectures

Lecture 1:

Physics: quantum interference, Born probability rule, configuration space

Mathematics: Laplace operator, many particle Schroedinger eq as a PDE, probability densities (without technical measure theory)

Lecture 2:

Physics: quantum formalism, elements of Copenhagen interpretation

Mathematics: matrix, linear operator on a function space, eigenvalue and eigenvector, inner product, self-adjoint matrices as observables, orthonormal basis

Lecture 3:

Physics: examples for application of quantum formalism, assumption of completeness of the quantum formalism

Mathematics: Hilbert space (without topology), square-integrable function, unitary matrices, differential operators, analogy with matrices, examples of non-commuting operators

Lecture 4:

Physics: abstract Born probability rule, more about Copenhagen interpretation

Mathematics: unitarity of Schroedinger eq (heuristically), projection operator, spectral theorem for self-adjoint matrices, Fourier transform, Heisenberg uncertainty relation

Lecture 5:

Physics: spin, Stern-Gerlach experiment

Mathematics: multi-component wave functions, spinor, connection with rotation group, matrix operators, Pauli matrices, Pauli eq

Lecture 6:

Physics: concept of hidden variables, more about the completeness assumption, commuting observables

Mathematics: commuting families of operators, joint spectrum, concept of random variable (discrete or continuous), marginal/conditional probability distribution (discrete case)

Lecture 7:

Physics: hidden variables and the relevance of maps from observables to random variables

Mathematics: Limitations on maps between noncommutative and commutative structures: von

Neumann's theorem, Gleason's theorem, the paradox of Kochen and Specker

Lecture 8:

Physics: entanglement in many-particle quantum mechanics

Mathematics: many-particle Schroedinger eq. When do product solutions exist? tensor product of Hilbert spaces (in finite dimension), of vectors, of operators, of orthonormal bases

Lecture 9:

Physics: More about hidden variables

Mathematics: Bell's inequality, Hardy's theorem, the theorem of spectral incompatibility

Lecture 10:

Physics: the paradox of Einstein, Podolsky and Rosen (EPR)

Lecture 11:

Physics: Bell's proof of quantum nonlocality, the EPR argument with spin

Mathematics: review of Bell's inequality and Pauli matrices, logical structure of Bell's argument

Lecture 12:

Physics: density matrix and mixed quantum state

Mathematics: trace, positive operator, von Neumann eq

Lecture 13:

Physics: Schroedinger's cat and the measurement problem

Mathematics: many-particle Schroedinger eq, consequences of linearity

Lecture 14:

Physics: reduced density matrix, decoherence and the measurement problem

Mathematics: partial trace

Lecture 15:

Physics: Bohmian mechanics, examples (interference)

Mathematics: Systems of ODEs, vector field on configuration space, continuity equation, transport of probability, equivariance

Lecture 16:

Physics: predictions of Bohmian mechanics

Mathematics: support of a function, law of large numbers

Lecture 17:

Physics: spin in Bohmian mechanics, nonlocality in Bohmian mechanics, hidden variables in Bohmian

mechanics

Mathematics: positive-operator-valued measures

Lecture 18:

Physics: Nelson's stochastic mechanics, modal interpretations

Mathematics: concept of Markov chain, Markov process, overview of diffusion processes

Lecture 19:

Physics: decoherent/consistent histories (Gell-Mann, Hartle, Griffiths, Omnes), linearly positive histories

Mathematics: probability consistency conditions

Lecture 20:

Physics: Everett's interpretation, Schroedinger's first theory, Bell's version of Everett; wave function monism vs. primitive ontology

Lecture 21:

Physics: wave function collapse as a physical process (Wigner, von Neumann, Penrose, Leggett), the measurement problem

Lecture 22:

Physics: the spontaneous collapse model of Ghirardi-Rimini-Weber (GRW)

Mathematics: Markov chain in continuous time, jump rate, point process, Poisson process (heuristically)

Lecture 23:

Physics: GRW theory, primitive ontology, predictions, examples

Mathematics: positive-operator-valued measure

Lecture 24:

Physics: classical limit of quantum mechanics

Mathematics: Ehrenfest theorem, scaling limit, Hamilton-Jacobi eq, local plane waves

Lecture 25:

Physics: outlook on quantum field theory

Mathematics: Fock space, configuration space of a variable number of particles, quantum fields

Lecture 26:

Physics: outlook on quantum cosmology, wave function of the universe

Mathematics: Riemannian metric on R^3 , diffeomorphism (heuristically), superspace of all Riemannian 3-geometries, Wheeler-de Witt eq