Assignment 9

Turn in starred problems Thursday, December 10.

Exercises:

Chapter 6: *1, *2, 3, 4, *7, 8, 10, 11

Remarks, instructions, hints, and extra questions:

1. I will hold office hours on Wednesday, December 9, 11:00–12:00.

2. Our final exam is **Friday, December 18, 8:00–11:00 A.M. in Hill 124**. We will work out some office hours earlier in that week.

3. I have honored a student request not to collect too many problems from this assignment, but these are all important and you should work out all of them before the final.

4. The most important case of Exercise 10 is $p = q = 2$, when (as Rudin points out) the result in (c) is called the *Schwarz inequality*, although if f and g really take complex values it is more usual to write this in an equivalent form with an extra complex conjugation:

$$\left| \int_a^b f\,\overline{g}\,d\alpha \right| \le \left(\int_a^b |f|^2\,d\alpha \right)^{1/2} \left(\int_a^b |g|^2\,d\alpha \right)^{1/2}.$$

In this case the inequality can be proved just as we proved the Schwarz inequality in $\mathbb{C}^n$: observe (with the notation of Exercise 11) that

$$\|f - \lambda g\|^2 = \int_a^b |f - \lambda g|^2\,d\alpha \ge 0$$

for all $\lambda \in \mathbb{C}$, and make an appropriate choice of λ. Note that in $\mathbb{C}^n$ we used the notation $\langle u, v \rangle = \sum_{i=1}^n u_i \overline{v}_i$; the corresponding quantity now is

$$\langle f, g \rangle = \int_a^b f\,\overline{g}\,d\alpha,$$

and the Schwarz inequality can again be written as

$$|\langle f, g \rangle| \le \|f\|\,\|g\|.$$