COMPACTNESS

For $A \subset X$, with X an arbitary metric space:

Property P of A	$P \Rightarrow A$ compact?	A compact $\Rightarrow P$?
A is closed	No	Yes
A is closed and $A \subset K$, K compact	Yes	Uninteresting
A is limit point compact	Exercise 2.26	Yes
Every family $\mathcal{F}$ of closed subsets of A with the finite intersection property satisfies $\bigcap_{F \in \mathcal{F}} F \neq \emptyset$.	Exercise	Yes

For $A \subset \mathbb{R}^k$, $k \in \mathbb{N}$:

Property P of A	$P \Rightarrow A$ compact?	A compact $\Rightarrow P$?
A is closed	No	Yes
A is closed and $A \subset K$, K compact	Yes	Uninteresting
A is limit point compact	**Yes**	Yes
Every family $\mathcal{F}$ of closed subsets of A with the finite intersection property satisfies $\bigcap_{F \in \mathcal{F}} F \neq \emptyset$.	Exercise	Yes
A is a k-cell	Yes	Uninteresting
A closed and bounded	Yes (The Heine-Borel Theorem)	Yes

1. A "Yes" in the table indicates a result which we proved in class or which follows immediately from such a result; a "No" indicates that we proved, or it is obvious, that the corresponding implication is false.

2. Note that in passing from the first to the second table we added two lines and changed one existing entry.

3. The fact that a k-cell in $\mathbb{R}^k$ is compact follows of course from the Heine-Borel Theorem; I have included it as a separate line in the second table because we proved it first, as part of our proof of Heine-Borel.