

## WRITING PROOFS

**Content—the logic of the proof**

- A proof should contain a clear and rigorous chain of reasoning leading from the hypothesis to the conclusion. You should give arguments supporting all the steps. (More precisely, give arguments for all the steps except for those which are “obvious”. Learning what this means—i.e., learning how much to write down for each argument—is part of learning the art of writing proofs). Everything written down should be relevant to this chain of reasoning: don’t start by writing down a list of things you know, and don’t digress as you go along.
- Do not work backward from the desired conclusion to the given hypotheses; this may be helpful at the preliminary stage of figuring out how to construct a proof, but the final version must move from what you know to what you want to establish, to be sure that the logic works in that direction. Another way to say this is that you should never write down a statement or equality unless you *know* that it is true—from definitions or previously established results, from the hypotheses which are given, or from some chain of reasoning based on these—or unless you state explicitly that it has *not* been established (for example, you might remark that “We must show that . . . ,” or a proof by contradiction might begin “We proceed by contradiction; suppose then that . . . .”)
- Proof by contradiction is a valuable tool, but it should be used sparingly. Often such a proof is really a direct proof in disguise; removing the disguise simplifies the presentation.
- When you use a result from the book or class, say so explicitly, and indicate that you are aware of and have checked the hypotheses. For example, “Since  $f$  and  $g$  are continuous, Lemma 6.66 implies that . . . .” Of course, sometimes verifying the hypotheses involves a lot of explicit work, after which you might say “Hence by a theorem proved in class, . . . .”
- When you are done, read your proof critically. Pretend that it was written by a stranger, and that you did not know what he or she was thinking. Does the proof then convince you absolutely? If not, try again.

**Style—the language of the proof**

- Write in complete, grammatically proper sentences. Remember that the equal sign is a verb. Avoid dangling modifiers.
- Study proofs in the text, in other books, and from the lectures, to get a feeling for good mathematical style.
- Don’t use the notations  $\forall$  and  $\exists$ . Don’t introduce mysterious abbreviations to save writing out words.

**Writing up homework**

- Don’t turn in scratch work. Once you have decided how to construct a proof, write out the details neatly to turn in.

**Follow up**

- When your work is returned to you, read the comments and be sure you understand their point. If you don’t, come to see me to talk about them.