Assignment 1

Turn in starred problems Wednesday, January 28

Exercises:

Chapter 7: 1, 2, 3, *4, 5, 6, *7, *8, *9

Remarks, hints, and extra questions:

5, 6: There is no connection between absolute convergence and uniform convergence, and these two problems point that out.

8. The function constructed here is called a *saltus* function, from the Latin *saltus*, a leap or jump. (The word *sauté* comes, through French, from the same root: food being sautéed jumps around in the pan.) Intuitively one might say that the function f is constant everywhere except at the points x_n, where it has a jump discontinuity. But the sequence $\{x_n\}$ might be an enumeration of the rationals, in whch case the jump points are dense and it is hard to understand just where f is constant!
Prove also: f is continuous at x_n if and only if $c_n = 0$.

9. The last question here is somewhat unclear. Probably what Rudin means by "the converse" is this:
Proposition: If $\{f_n\}$ is a sequence of continuous functions defined on a metric space E, f is a function defined on E, and $f_n(x_n) \to f(x)$ for every $x \in E$ and every sequence $\{x_n\}$ of points in E with $x_n \to x$, then $\{f_n\}$ converges uniformly to f.
Prove that this proposition is not true in general, but holds if E is compact.