Assignment 2

Due Wednesday, February 4

Exercises:

Chapter 7: 10, *12, *14, 15, *16, 18, *19

Remarks, hints, and extra questions:

10. This is rather similar to Exercise 8 of this chapter. Theorem 7.16 is relevant.

12. Exercises 7 and 8 of Chapter 6 define

$$\int_0^\infty g(x)\,dx = \lim_{\substack{a\to 0\\ b\to\infty}} \int_a^b g(x)\,dx. \tag{4.1}$$

The statement in the problem that

$$\int_0^\infty g(x)\,dx < \infty$$

just says the double limit in (4.1) exists (remember that $g \geq 0$). You will have to verify that the relevant improper integrals of f_n and f exist.

The comment at the end of the exercise is worth reading. The point of the problem as stated is to extend Theorem 7.16 to improper integrals, under the assumption of the existence of the "dominating" function $g(x)$. Uniform convergence is still assumed. On the other hand, the note refers to an extension to the case in which the convergence is pointwise. This would be of interest even for (proper) Riemann integrals over a finite interval $[a, b]$. For example, this result would have completed our proof that the sequence $\{\sin nx\}$ has no convergent subsequence.

14. Clever people can come up with amazing examples!

16. You can mimic the proof of Theorem 7.25(a). The result can also be deduced from the Ascoli-Arzela Theorem (Theorem 7.25(b)), if you con't want to get your hands dirty.

19. We will talk about these ideas in class on Monday 2/2, so you might wait to think about this one until after that.

*Turn in starred problems Wednesday 2/4.