Assignment 8

Due Wednesday, April 1

Exercises:

Chapter 9: 7, 8*, 9*, 11, 13, 14*, 15

8.A* Suppose that $\mathbf{f} : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $\mathbf{f}(x_1, x_2) = (x_1(x_1 + x_2), x_2)$.
(a) Prove *directly from the definition* that $\mathbf{f}$ is differentiable at each point of $\mathbb{R}^2$. This means: find $\mathbf{f}'(\mathbf{x})$ and prove that it satisfies

$$\lim_{\mathbf{h}\to 0} \frac{1}{|\mathbf{h}|} \left|\mathbf{f}(\mathbf{x}+\mathbf{h}) - \mathbf{f}(\mathbf{x}) - \mathbf{f}'(\mathbf{x})\mathbf{h}\right| = 0.$$

(b) Find explicitly the best affine approximation $\mathbf{f}(\mathbf{x}) \approx \mathbf{a} + A\mathbf{x}$ near the point (3,2).

Extra credit: Chapter 9, problem 12

Remarks and hints:

9. The proof was sketched in class; just fill in the details.

14, 15. Ones intuition about the behavior of functions of several variables is often wrong. It's good to have a few examples as a corrective. Note that the result of problem 7 is required in 14.

A. This is a simple problem, just to check your understanding of the definition of differentiability.

12. This is a very geometric problem. I don't think it is particularly important for you, but I thought that you might have fun playing with it, and that if you played successfully you shold get some extra credit.

*Turn in starred problems Wednesday 4/7,