Assignment 9
Due Wednesday, April 8

Exercises:

Chapter 9: 16, 17*, 21*

9.A* Suppose that $f : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $f(x,y) = (x^2y^2, x^2+y^2)$.

(a) Find, from the inverse function theorem, those points $(x,y) \in \mathbb{R}^2$ such that f has a continuously differentiable inverse in a neighborhood of (x,y).

(b) Given that (1,2) is such a point, and noting that $f(1,2) = (4,5)$, find the affine transformation $g : \mathbb{R}^n \to \mathbb{R}^n$ which yields the best approximation to f^{-1} in the neighborhood of $(4,5)$ (in the sense of used in the definition of differentiability).

9.B* Consider the equations $e^{xy} + zx = 5$, $x^2 + y^2 + z^2 = 8$. Show that a local $\mathcal{C}^1$ solution for y and z in terms of x exists near the point $x = 2$, $y = 0$, $z = 2$, and compute the gradient of this solution at $y = 0$, $z = 2$.

(b) Does a local solution for x and z in terms of y exist near this point? For x and y in terms of z?

Remarks and hints:

We have not discussed determinants in class, but I think you are aware that a (square) matrix is invertible if and only if its determinant is nonzero. This is usually the way to test invertibility.

17. The *Jacobian* of $\mathbf{f}$ at $\mathbf{x}$ is $J_{\mathbf{f}}(\mathbf{x}) \equiv \det \mathbf{f}'(\mathbf{x})$. Thus the statement that "the Jacobian of f is not zero at any point of $\mathbb{R}^2$" just means that $\mathbf{f}'(\mathbf{x})$ is invertible for all $\mathbf{x}$. Those who have studied complex variables might note that this f is the map $f : \mathbb{C} \to \mathbb{C}$ given by $f(z) = e^z$.

A,B. These are not supposed to be difficult—just straightforward use of the inverse and implicit function theorems.

*Turn in starred problems Wednesday 4/8,