

$$\omega = \sum_{i_1, \dots, i_k} a_{i_1 i_2 \dots i_k}(\mathbf{x}) dx_{i_1} \wedge dx_{i_2} \wedge \dots \wedge dx_{i_k} \Leftrightarrow$$

$$\int_{\varphi} \omega = \int_{J^k} \sum_{i_1, \dots, i_k} a_{i_1 i_2 \dots i_k}(\varphi(\mathbf{t})) \frac{\partial(x_{i_1}, \dots, x_{i_k})}{\partial(t_1, \dots, t_k)} d\mathbf{t}.$$


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$$\omega = \sum_I b_I dx_I \in \mathcal{E}_k(E), \quad \kappa = \sum_{I'} c_{I'} dx_{I'} \in \mathcal{E}_l(E), \quad \Rightarrow$$

- $\omega \wedge \kappa = \sum_{I, I'} b_I(\mathbf{x}) c_{I'}(\mathbf{x}) dx_{i_1} \wedge \dots \wedge dx_{i_k} \wedge dx_{i'_1} \wedge \dots \wedge dx_{i'_l}$
- $d\omega = \sum_I (db_I) \wedge dx_I = \sum_I \sum_{i=1}^n (D_i b_I) dx_i \wedge dx_I.$
- $\omega \wedge \kappa = (-1)^{kl} \kappa \wedge \omega$
- $d(\omega \wedge \kappa) = (d\omega) \wedge \kappa + (-1)^k \omega \wedge (d\kappa)$

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$$T : E \rightarrow V, \quad T(\mathbf{x}) = (\mathbf{t}_1(\mathbf{x}), \mathbf{t}_2(\mathbf{x}), \dots, \mathbf{t}_m(\mathbf{x})), \quad \omega = \sum_I a_I(\mathbf{y}) dy_I \in \mathcal{E}_k(V) \quad \Rightarrow$$

- $\omega_T = \sum_{0 \leq i_1 \leq \dots \leq i_k \leq 1} a_{i_1 \dots i_k}(T(\mathbf{x})) dt_{i_1} \wedge \dots \wedge dt_{i_k}$
- $(\omega \wedge \kappa)_T = \omega_T \wedge \kappa_T \quad \text{if } \kappa \in \mathcal{E}_l(V)$
- $d(\omega_T) = (d\omega)_T$
- $\int_{\varphi} \omega_T = \int_{T \circ \varphi} \omega$

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- $\psi_{i\alpha}(s_i, \dots, s_{k-1}) = (s_1, \dots, s_{i-1}\alpha, s_i, \dots, s_{k-1})$
- $\partial\varphi = \sum_{i=1}^k \sum_{\alpha=0,1} (-1)^{i+\alpha} \varphi \circ \psi_{i\alpha}^k$