Answers to some problems for the 1st midterm problem set

8. Answer:

$$\begin{aligned} y_1(t) &= 1-(3-2e^{t-3})H(t-3), \\ y_2(t) &= 30(1-e^{t-3})H(t-3). \end{aligned} \tag{1}$$

9. Taking the Laplace, get

$$(s^2-3s+2)Y(s) = \frac{4}{s^2} + s - 4 = \frac{4+s^3-4s^2}{s^2}.$$

$$Y(s) = \frac{s^3-4s^2+4}{s^2(s-2)(s-1)}.$$

Partial fractions,

$$\frac{s^3-4s^2+4}{s^2(s-2)(s-1)} = \frac{A}{s-2} + \frac{B}{s-1} + \frac{C_1}{s} + \frac{C_2}{s^2}.$$

Solve for coefficients:

$$A=-1, \quad B=-1, \quad C_1=3, \quad C_2=2.$$

Inverse Laplace,

$$y(t) = 3+2t-e^{2t}-e^t.$$

10. The function f is equal to zero for $0 < t < 2$, then $f(t) = 1$ for $2 \le t \le 5$, then $f(t) = 0$ for $5 \le t \le 6$, and finally, $f(t) = 4$ for $t > 6$. The Laplace transform is

$$F(s) = \frac{e^{-2s}-e^{-5s}+4e^{-6s}}{s}.$$

11. (a) We can write

$$(\alpha, 0, \beta+2\alpha, -3\beta) = \alpha(1,0,2,0) + \beta(0,0,1,-3).$$

Define vectors $\mathbf{F}_1 = (1,0,2,0)$ and $\mathbf{F}_2 = (0,0,1,-3)$. The span the subspace, and they are linearly independent (because the linear combination above can only be made zero by taking $\alpha = \beta = 0$. Therefore, $\mathbf{F}_1$ and $\mathbf{F}_2$ are a basis.

(b) The dimension is 2.

(c)

$$\cos\theta = \frac{(\mathbf{F}_1 \cdot \mathbf{F}_2)}{||\mathbf{F}_1||\,||\mathbf{F}_2||} = \frac{2}{\sqrt{5}\sqrt{10}}.$$