

# Sample problems for exam #1 in Math 421(1)

The exam will cover Laplace transform and Sections 5.4, 5.5, 6.1. On the exam, you will be expected to show all the steps. A bare answer is not sufficient. The exam will be shorter than this set of problems!

Some of the solutions for problems 1–7 and 12–16 can be found at

[http://www.math.rutgers.edu/~greenfie/mill\\_courses/math421/rev1\\_ans.html](http://www.math.rutgers.edu/~greenfie/mill_courses/math421/rev1_ans.html)

These problems are the same as in Section 2 of the course taught by Prof. Greenfield. The rest of the problems are different. The answers will be posted.

1. Find the inverse Laplace transform of

$$\frac{se^{-s}}{s^2 - 1} - \frac{s^2}{(s + 2)^2}$$

2. Compute the convolution  $f(t) = (H(t)t^2) * (H(t)\cos(t))$  and its Laplace transform.
3. Use the Laplace transform to solve the initial value problem  $y'' + 2y' - 3y = H(t - 2)(t - 1)$  with  $y(0) = 1$  and  $y'(0) = -1$ .

4. Consider the function  $f(t) = \begin{cases} 0 & 0 < t < 1 \\ t & 1 < t < 3 \\ 1 & 3 < t \end{cases}$

- a) Sketch  $f(t)$ .
- b) Express  $f(t)$  in terms of Heaviside functions.
- c) Calculate the Laplace transform of  $f(t)$ .

5. Find the inverse Laplace transform of

$$\frac{s}{(s^2 + 36)^2}$$

6. Solve for  $y(t)$  where  $y(t) = 1 - \int_0^t y(\tau)e^t e^{-\tau} d\tau$ .

7. Evaluate  $\int_0^\infty (\delta(t - e)\ln(t) + \delta(t - \pi)H(t - 2)) dt$ .

8. Solve the system,

$$\begin{aligned}y_1' + 2(y_1 - y_2) &= 10\delta(t - 3), \\y_2' + 3(y_1 - y_2) &= 0,\end{aligned}$$

with the initial conditions,  $y_1(0) = y_2(0) = 0$ .

9. Solve

$$y'' - 3y' + 2y = 4t, \quad y(0) = 1, \quad y'(0) = -1.$$

10. Plot the function  $f(t) = H(t - 2) - H(t - 5) + 4H(t - 6)$ . Find its Laplace transform.

11. Suppose that  $V$  is a subspace of  $\mathbf{R}^5$  which consists of vectors,

$$(\alpha, 0, \beta + 2\alpha, -3\beta), \text{ for all } \alpha \text{ and } \beta.$$

(a) Find a basis of  $V$ .

(b) What is the dimension of  $V$ ?

(c) For each pair of vectors in the basis, find the cosine of the angle between the vectors.

12. Are the vectors  $(0, 1, 3, 4)$ ,  $(1, 2, 4, -1)$ , and  $(-2, 1, 1, 0)$  linearly independent in  $\mathbf{R}^4$ ?

13. Is  $(1, 1, 1, 1, 1)$  in the span of the vectors  $(0, 2, 0, 2, 1)$ ,  $(1, 0, 1, 0, 1)$  and  $(2, 2, 2, 2, 2)$  in  $\mathbf{R}^5$ ?

14. Give an example of a subspace of  $\mathbf{R}^7$  of dimension 4. What is a basis of your example?

15. If  $A = \begin{pmatrix} 3 & 0 & -1 \\ 1 & 2 & 2 \\ 4 & 2 & 1 \end{pmatrix}$  and  $B = \begin{pmatrix} 3 & 2 & 0 \\ 2 & -1 & 1 \end{pmatrix}$  compute any of the matrices following which are defined, and explain why the others are *not* defined.

$$A^2 \quad AB \quad BA \quad B^2 \quad A^2 + 3A \quad A + B$$

16. Use the definition of Laplace transform to find the Laplace transform of  $t$ .