

# Sample problems for Midterm 1 make-up

**Heaviside function.** Review Def 3.5, Figure 3.14. Solve Section 3.3, problems 8, 9, 10.

**Dirac delta-function.** Review Theorem 3.12. Solve Section 3.5, problems 7 and 8.

**Shifting theorems.** Solve Section 3.3, problems 16, 19, 20. Also, find the inverse transform of

$$\frac{3e^{-5s} + 7}{s^2 - 3s + 2}.$$

Hint: first write as a sum of two terms, one containing the exponent. Do partial fractions for the function,

$$\frac{1}{s^2 - 3s + 2}.$$

Then use a shifting theorem where needed.

**Convolution.** Review Def. 3.6 and Theorem 3.9. Do Section 3.4, problems 3, 6.

Also, find the convolution of functions

$$f(t) = 2t, \quad g(t) = e^{-3t}.$$

Then find the Laplace transform of  $f * g$  by two methods: directly and by using Theorem 3.9.

Also, find the inverse transform of

$$\frac{3e^{-5s}}{s(s^2 - 3s + 2)}.$$

Hint: Use the result for  $\mathcal{L}^{-1}\left(\frac{1}{s^2 - 3s + 2}\right)$  from above, and calculate

$$\mathcal{L}^{-1}\left(\frac{1}{s(s^2 - 3s + 2)}\right)$$

by the convolution method. Then use a shifting theorem.

**Solution of ODE's.** Do Section 3.3, problems 37, 39.