

## Solution for Quiz 2

Find the Laplace transform of the function  $f(t)$  given by

$$f(t) = \begin{cases} 0, & 0 \leq t < 8, \\ 5t, & t \geq 8. \end{cases}$$

First we express this function in terms of the Heaviside function,

$$f(t) = 5tH(t - 8).$$

Then we evaluate the Laplace transform:

$$\begin{aligned} \mathcal{L}[f] &= \mathcal{L}[5tH(t - 8)] = 5\mathcal{L}[H(t - 8)t] = 5\mathcal{L}[H(t - 8)(t - 8 + 8)] = \\ &5\mathcal{L}[H(t - 8)(t - 8) + 8H(t - 8)] = 5\mathcal{L}[H(t - 8)(t - 8)] + 40\mathcal{L}[H(t - 8)]. \end{aligned}$$

From Second Shifting Theorem,

$$\mathcal{L}[H(t - 8)(t - 8)] = e^{-8s} \mathcal{L}[t] = \frac{e^{-8s}}{s^2}.$$

From the same theorem, or from Example 3.11 of the book, we have

$$\mathcal{L}[H(t - 8)] = e^{-8s} \mathcal{L}[1] = \frac{e^{-8s}}{s}.$$

Therefore,

$$\mathcal{L}[f] = 5\frac{e^{-8s}}{s^2} + 40\frac{e^{-8s}}{s}.$$