

Solution for Quiz 3

- Find

$$\mathcal{L}^{-1} \left[\frac{1}{(s+1)^3} \right].$$

First shifting theorem:

$$\mathcal{L}^{-1} [F(s-a)] = e^{at} \mathcal{L}^{-1} [F(s)].$$

Here $a = -1$, and $F(s-a) = 1/(s+1)^3$, so that $F(s) = 1/s^3$. We have $\mathcal{L}^{-1} [1/s^3] = t^2/2$. Therefore,

$$\mathcal{L}^{-1} \left[\frac{1}{(s+1)^3} \right] = \frac{e^{-t}}{2} t^2.$$

- Check that

$$\mathcal{L}[tf(t)] = -\frac{d}{ds} \mathcal{L}[f(t)]$$

with $f(t) = t$. The left hand side is

$$\mathcal{L}[tf(t)] = \mathcal{L}[t^2] = 2/s^3.$$

The right hand side is

$$\begin{aligned} -\frac{d}{ds} \mathcal{L}[f(t)] &= -\frac{d}{ds} \mathcal{L}[t] = \\ &= -\frac{d}{ds} \left(\frac{1}{s^2} \right) = -\frac{-2}{s^3} = \frac{2}{s^3}. \end{aligned}$$

This is equal to the left hand side.