Some solutions for Homework 3

- 3.6: 6. The answer is

$$\frac{-6+6e^{5t}-30t-75t^2-125t^3}{2750}.$$

- 3.6: 10. We have

$$Y(s)=\frac{F(s)}{(s+4)(s+6)}+\frac{10+s}{(s+4)(s+6)}.$$

$$\mathcal{L}^{-1}\left[\frac{1}{(s+4)(s+6)}\right]=-\frac{1}{2}e^{-6t}+\frac{1}{2}e^{-4t}.$$

Next,

$$\frac{10+s}{(s+4)(s+6)}=\frac{3}{s+4}-\frac{2}{s+6},$$

and

$$\mathcal{L}^{-1}\left[\frac{s+10}{(s+4)(s+6)}\right]=3e^{-4t}-2e^{-6t}.$$

Therefore, we get

$$y(t)=f*(-\frac{1}{2}e^{-6t}+\frac{1}{2}e^{-4t})+3e^{-4t}-2e^{-6t}.$$

- 3.4: 18. The equation is

$$f=-t+f*\sin t.$$

Therefore,

$$F=-\frac{1}{s^2}+F\left(\frac{1}{s^2+1}\right).$$

$$F(s)=-\frac{s^2+1}{s^4}=-\frac{1}{s^2}-\frac{1}{s^4}.$$

$$f(t)=-t-t^3/6.$$

- 3.5: 2.

$$(13 - 4s + s^2)Y = 4e^{-3s}.$$

$$Y(s) = \frac{4e^{-3s}}{s^2 - 4s + 13} = \frac{4e^{-3s}}{(s-2)^2 + 9}.$$

$$\mathcal{L}^{-1}\left[\frac{4e^{-3s}}{(s-2)^2+9}\right] = \mathcal{L}^{-1}\left[\frac{4e^{-3(s-2)e^6}}{(s-2)^2+9}\right] = 4e^6 e^{2t}\mathcal{L}^{-1}\left[\frac{e^{-3s}}{2^2+9}\right],$$

by first shifting theorem. Also,

$$\mathcal{L}^{-1}\left[\frac{e^{-3s}}{2^2+9}\right] = \frac{1}{3}\sin[3(t-3)]H(t-3),$$

by second shifting theorem. Therefore,

$$y(t) = \frac{4e^{6+2t}}{3}\sin[3(t-3)]H(t-3).$$