Section 4.1  Do 4.1 and 4.6.
You should look at and understand all the other exercises in this section. We discuss
them here, in turn.

We discussed exercises 4.2 in class; one could prove this statement by noting that if
there were a bridge $uv$ in a graph $G$ all of whose vertice have even degree and if $G_1$ were
the component of $G - uv$ containing $u$, then $u$ would be the single vertex of odd degree in
$G_1$. Indeed, $\deg_{G_1}(u) = \deg_G(u) - 1$ would be odd, and $\deg_{G_1}(x) = \deg_G(x)$ would be even
for all other vertices $x$ in $G_1$. But this is impossible, so $G$ cannot contain a bridge if all the
vertices of $G$ have even degree.

We essentially did exercises 4.3 and 4.4 in our class discussion of bridges, but review the
statement and arguments to make sure you understand them. Exercise 4.5 is trivial once
you realize that $G$ in (a) is a tree and $G$ in (b) is a forest, since one then just applies what
we know about the number of edges in a tree or in a forest.

Exercise 4.6 may be rephrased as: suppose $G$ is a graph of order $n \geq 3$ without bridges,
but for every edge $e$ of $G$, $G - e$ is a tree. What is $G$? Hint: Try drawing a graph with the
required properties.

Section 4.2.  • 4.7 For this problem “draw all trees” means “draw all trees up to isomor-
phism.” For part (a) you can use the list we developed of all graphs of order 5.
• 4.9, 4.10, 4.12, 4.14, 4.15, 4.16, 4.19, 4.20.

Again, you should look at all the exercise statements, even those not assigned, and study
the solutions at the end of the text to the odd numbered problems.

Section 4.3  4.25, 4.27, 4.29.

5A. (a) Show that if $n \geq 3$ and if $d_1 \geq \cdots \geq d_n$ is a sequence of positive integers such that
$\sum_{i=1}^n d_i = 2(n - 1)$, then $d_1 > 1$ and $d_{n-1} = d_n = 1$.

(b) The aim of this problem is to prove the claim: if $(d_1, \ldots, d_n)$ is a sequence of positive
integers such that $\sum_{i=1}^n d_i = 2(n - 1)$, then it is the degree sequence of a tree. This can be
proved by induction on the order $n$ of the tree.

(i) Prove the statement for $n = 2$.

(ii) Prove the induction step. Assume that the claim is $n = k$, where $k \geq 2$. We want to
show that this implies that the claim is true for $n = k + 1$. Thus let $(d_1, \ldots, d_{k+1})$ be
a sequence of positive integers such that

$$\sum_{i=1}^{k+1} d_i = 2((k + 1) - 1).$$

(Assignment continues on second page.)
Without loss of generality, assume that $d_1 \geq d_2 \geq \cdots d_{k+1}$. Consider also the sequence of length $k$. From (a) we know that $d_1 > 1$ and $d_{k+1} = 1$. Use the induction hypothesis on $(d_1 - 1, d_2, \ldots, d_k)$ to show that $(d_1, \ldots, d_{k+1})$ is the degree sequence of a tree.

Write out the induction proof following these steps carefully. (Or, if you have a different and possibly better proof, you can submit that!)

Hand in 4.6, 4.14, 4.29, and 5A on October 11.