Problem set 8, Math 428, Fall 2007

Section 5.4: 5.33, 5.34, 5.35, 5.36, 5.37

9A. For the graph below:
Find the connectivity, $\kappa(G)$, and edge-connectivity, $\lambda(G)$, for the following graph. Justify your answer fully. In the justification, you can use Whitney’s theorem ($\kappa(G) \leq \lambda(G) \leq n - 1$) and/or Menger’s theorem either for vertices or edges, or other results established in class or in the text about connectivity.

9B. Let $x$ be the leftmost vertex of the graph below and let $y$ be the rightmost vertex. Find the maximum number of internally disjoint $(x,y)$-paths and justify your answer using Menger’s theorem for vertices.
Find the maximum number of edge-disjoint $(x,y)$-paths and justify your answer using Menger’s theorem for edges (see Theorem 5.22).

9C. (a) List all nonseparable graphs of orders 2, 3, 4, and 5.
(b) Suppose that we have a graph of order 7 that has three blocks. One way that the blocks may be structured, just in terms of possible intersection and block orders is to have all three intersect in one cut-vertex, and to have 2 vertices in one block, 3 in a second, and 4 in the third, as in the figure on the next page. List all the ways that the blocks intersections and orders can be arranged.
(c) How many graphs, up to isomorphism, are there of order 7, with three blocks and with 7 edges? (This problem is from *Graph Theory and its Applications* by Gross and Yellen, CRC Press, 2006. Hint: A tree on 7 vertices has 6 edges. How many cycles must a graph with 7 vertices and 7 edges have. Combine the information from (a) and (b) to pick out which types of combinations in (b) are possible. Then see how many ways to arrange them in non-isomorphic graphs.

9D. What are the vertex connectivity and the edge connectivity of the complete bipartite graph $K_{4,8}$?