Brook Taylor (1685-1731)

Graduated from Cambridge in 1702.
In 1708, theorem on center of oscillation – published in 1714
Johann Bernoulli disputed priority.
In 1715, *Methodus Incrementorum Directa et Inversa*
contained Taylor's Theorem (and difference method in calculus)

Spent his career refereeing the argument between Newton & Leibniz

George Berkeley 1685 - 1753

Irish philosopher
In 1734, wrote *The Analyst*

And what are these fluxions?
The velocities of evanescent increments. And what are these same evanescent increments? They are neither finite quantities, nor quantities infinitely small, nor yet nothing. May we not call them ghosts of departed quantities?

Colin Maclaurin (1698-1746)

Gaelic name "Cailean MacLabhruinn",
'Colin, the son of Laurence'.
Graduated University of Glasgow at age 14. (Divinity school after.)
Professor of Mathematics at age 19.

Maclaurin series are Taylor series expanded around 0, and are not attributed to Maclaurin due to the past discoveries, but still receive credit because of his use of them. In particular, he used these series to characterize maxima, minima, and points of inflection for infinitely differentiable functions.

In 1742 Maclaurin published his 2 volume *Treatise of fluxions*, the first systematic exposition of Newton's methods -- written as a reply to Bishop Berkeley's attack on the calculus for its lack of rigorous foundations.

Marquis Guillaume de l'Hôpital (1661-1704)

Author of the first European textbook on differential calculus,
l'Analyse des Infiniment Petits pour l'Intelligence des Lignes Courbes.

Published in 1696, the text includes the lectures of his teacher, Johann Bernoulli, in which Bernoulli discusses 0/0.

In 1694 he forged a deal with Johann Bernoulli. The deal was that l'Hôpital paid Bernoulli 300 Francs a year to tell him of his discoveries, which l'Hôpital described in his book. In 1704, after l'Hôpital's death, Bernoulli revealed the deal to the world.

The widespread story that l'Hôpital tried to get credit for inventing de l'Hôpital's rule is false: he published his book anonymously, acknowledged Bernoulli's help in the introduction, and never claimed to be responsible for the rule.

Maria Gaetana Agnesi (1718-1799)

The first important woman mathematician since Hypatia.
Born in Milan to a Math Professor (Pietro), eldest of 21 children.
"Walking Polyglot" – could speak 7 languages by age 11

1748: Instituzioni analitiche ad uso della gioventu italiana (Foundations of Analysis for the use of Italian Youth), first textbook to treat both Differential and Integral Calculus. Clear examples on Max/min problems, solids of revolution, improper integrals.

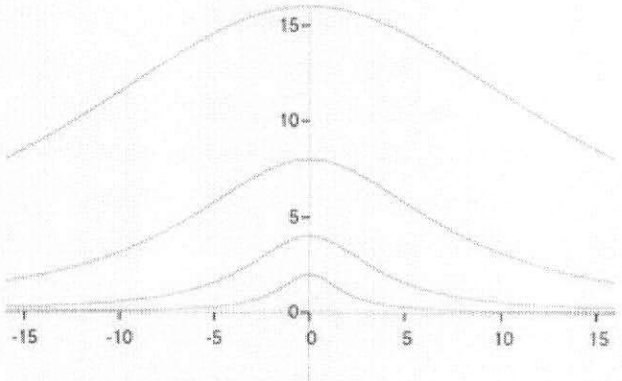
In 1750, on the illness of her father, she was appointed by the Pope (Benedict XIV) to the chair of Mathematics and "Natural Philosophy" at the University of Bologna.

Maria Agnesi's Witch

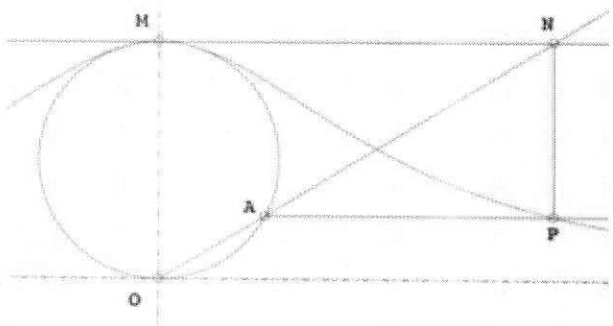
First studied by Fermat in 1666, it was an example in Agnesi's book.
English translator (John Colson of Cambridge)
mistook versiera (turning thing) as meaning
avversiera: woman contrary to God, or "witch"

The curve has equation $y = \dfrac{8a^3}{x^2 + 4a^2}$

It is drawn for a=1,2,4, 8 at right.



Derivation: Starting with a fixed circle, a point O on the circle is chosen. For any other point A on the circle, the secant line OA is drawn. The point M is diametrically opposite O. The line OA intersects the tangent at M at the point N. The line parallel to OM through N, and the line perpendicular to OM through A intersect at P. As the point A is varied, the path of P is the witch.

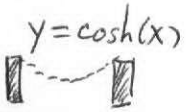


Johann Bernoulli (1667-1748) & Jakob Bernoulli (1654-1705)

Swiss brothers, lived in Basel.
J&J became familiar with Calculus through a correspondence with Leibniz, then collaborated on various applications. Later, Johann became jealous of Jakob's University position until 1705

Jacob is most famous for *Ars Conjectandi* (The Art of Conjecture), published after his death in 1713
(It deals with Probability Theory, application to "games of chance.")

In 1690: Jacob invented separation of variables to solve ODE's
In 1691, Jacob proposed, and Johann solved, the catenary problem.



In 1696 Johann Bernoulli proposed the Brachistone problem, despite already having solved the problem himself. This is the curve of fastest descent between two points – starting at rest and accelerating under gravity. Newton solved it in 12 hours, published in 1697.

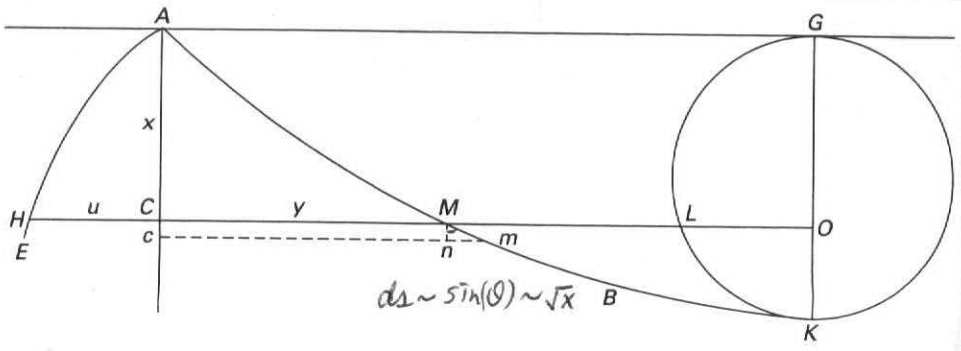


FIGURE 12.1 Johann Bernoulli's brachistochrone problem

Leonhard Euler (1707-1783)

Born in Basel, Switzerland, studied under Johann Bernoulli.
1726, got his Ph.D. On propogation of sound.
In 1727, Daniel Bernoulli convinced Catherine I to give
Euler a position at University of St. Petersburg. (Peter II)
1731 became chair of the Math Department there
In 1741, left for a more secure position in Berlin (25 years)
In 1766, returned to St. Petersburg until death (Catherine the Great)

Euler introduced the concept of a function and was the first to write $f(x)$ to denote the function f applied to the argument x.
He also introduced the modern notation for sin(x), cos(x).
• letter e for the base of the natural log was due to him,
• letter Σ for summations
• letter i to denote the imaginary number.
• Combined Newton and Leibniz approaches to Calculus
• The use of the Greek letter π to denote the ratio of circumference to radius was also popularized by Euler, although it did not originate with him.

Most famous textbooks:
• *Introductio in analysin infinitorum*, (*Intro to Analysis of the Infinite)* published in 1748,
• *Institutiones calculi differentialis* (*Methods of the Differential Calculus*) published in 1755

Branches of Math influenced by Euler:

• **Analysis**. Use of power series for exponential, arctan(x), sinh(x) and others such as

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \lim_{n\to\infty} \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} \right) = \frac{\pi^2}{6}.$$

•**Number Theory**: Euler Phi function, distribution of prime numbers

•**Topology and Graph Theory**: Euler characteristic of polyhedra, Seven Bridges of Konigberg

•**Differential Equations and Applied Math**: use of Fourier series, remainder for Maclaurin series

•**Physics**: Sound waves, Euler-Bernoulli Beam equation, orbits of comets

•**Logic**: Euler diagrams to illustrate reasoning by syllogisms