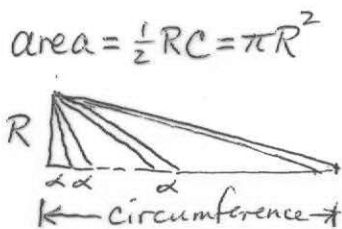
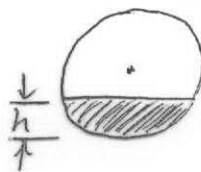


Indivisibles and Area

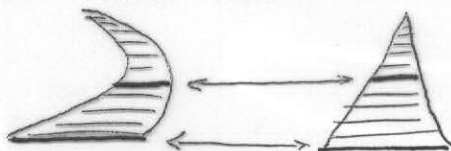
① Kepler 1615 (Wine Bottles)



How much wine is left in the barrel?

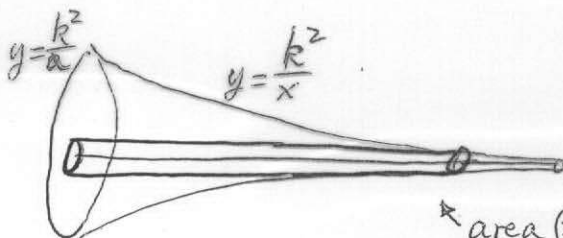


② Bonaventura Cavalieri 1635 and 1647



- Archimedes method of indivisibles
- Cavalieri's Principle in Nine Chapters (China ~200BC Han Dyn.)

③ Evangelista Torricelli 1643



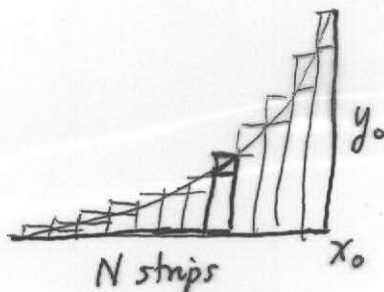
- surface area of shell $x=a$ constant, area of circle of radius $k\sqrt{x}$
- volume = (height) $A = \frac{k^2}{a} (2\pi k^2)$

area $(2\pi x) \left(\frac{k^2}{x}\right) = 2\pi k^2$

④ Fermat 1636 letter to Roberval (& vice versa)

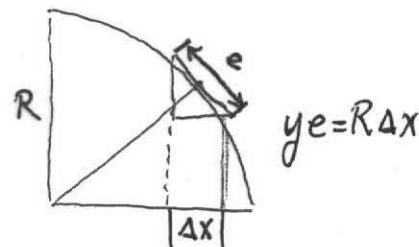
Area under $y = ax^n$

$$A = \frac{x_0 y_0}{n+1} = \frac{a}{n+1} x_0^{n+1}$$



⑤ Blaise Pascal 1657
Area under sine curve

$$\int_{\alpha}^{\beta} r \sin(\theta) d(\theta) = r [r \cos(\alpha) - r \cos(\beta)]$$



James Gregory (1638-1685)

Grew up, home-schooled and lived in Scotland. In 1663-1667 travelled to Flanders, Paris, Padua and Florence.

1667: wrote *Vera Circuli et Hyperbolae Quadratura*, in which he established the basic ideas of an infinitesimal analysis.

Using it, he showed how the areas of the circle and hyperbola could be obtained in the form of infinite power series,

1668: Gregory determined the power series expansions of the sine, cosine and tangent. He also established that

$$\int \sec x \, dx = \log(\sec x + \tan x)$$

solving a long standing problem in the construction of nautical tables.

In 1671, after seeing Barrow's book, Gregory established Taylor's Theorem (1715).



Grégoire de Saint-Vincent (1584-1667) Brussels, Belgium

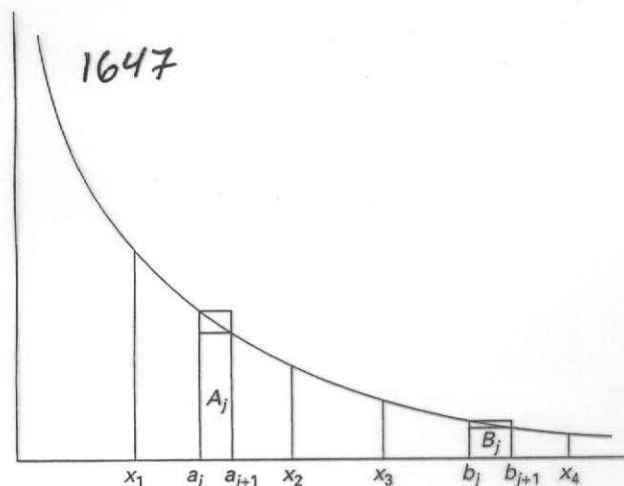


FIGURE 11.10 Gregory of St. Vincent's area under the hyperbola $xy = 1$



1668 Nicolas Mercator calculated

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

used Fermat-Roberval formulas for area