Leonhard Euler (1707-1783)

Born in Basel, Switzerland, studied under Johann Bernoulli.
1726, got his Ph.D. On propogation of sound.
In 1727, Daniel Bernoulli convinced Catherine I to give
Euler a position at University of St. Petersburg. (Peter II)
1731 became chair of the Math Department there
In 1741, left for a more secure position in Berlin (25 years)
In 1766, returned to St. Petersburg until death (Catherine the Great)

Euler introduced the concept of a function and was the first to write *f*(*x*) to denote the function *f* applied to the argument *x*.
He also introduced the modern notation for sin(x), cos(x).

- letter *e* for the base of the natural log was due to him,
- letter Σ for summations
- letter *i* to denote the imaginary number.
- Combined Newton and Leibniz approaches to Calculus
- The use of the Greek letter π to denote the ratio of circumference to radius was also popularized by Euler, although it did not originate with him.

Most famous textbooks:

- *Introductio in analysin infinitorum*, (*Intro to Analysis of the Infinite)* published in 1748,
- *Institutiones calculi differentialis* (*Methods of the Differential Calculus*) published in 1755

Branches of Math influenced by Euler:

- **Analysis.** Use of power series for exponential, arctan(x), sinh(x) and others such as

$$\sum_{n=1}^{\infty}\frac{1}{n^2}=\lim_{n\to\infty}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}\right)=\frac{\pi^2}{6}.$$

- **Number Theory**: Euler Phi function, distribution of prime numbers
- **Topology and Graph Theory**: Euler characteristic of polyhedra, Seven Bridges of Konigberg
- **Differential Equations and Applied Math**: use of Fourier series, remainder for Maclaurin series
- **Physics**: Sound waves, Euler-Bernoulli Beam equation, orbits of comets
- **Logic**: Euler diagrams to illustrate reasoning by syllogisms

From Figure 4.17, we can make some predictions about the behavior of unforced solutions. If ω is very large, then the coefficient $a = 1/(2 - \omega^2)$ is small. In this case solutions of the forced equation

$$\frac{d^2y}{dt^2} + 2y = \cos \omega t$$

are very close to solutions of the unforced equation. A large value of ω means the external force has very short period and large frequency. The above discussion predicts that such an external force with large frequency has only a small effect on solutions. The harmonic oscillator does not have enough time to respond to a push in one direction before the sign of the external force changes. A typical example is shown in Figure 4.18.

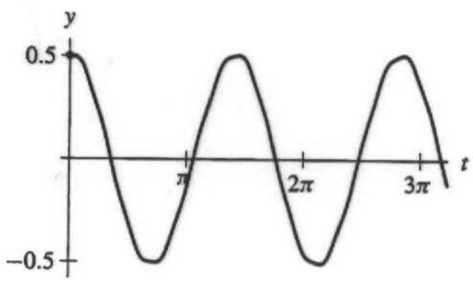

Figure 4.18
Solution of

$$\frac{d^2y}{dt^2} + 2y = \cos 10t$$

for initial conditions $y(0) = 0.5$, $y'(0) = 0$.

Qualitative behavior

Setting $\omega = \sqrt{2}$, the differential equation becomes

$$\frac{d^2y}{dt^2} + 2y = \cos \sqrt{2}\, t.$$

We can predict the behavior of solutions of this equation in two ways. First, we can study solutions of the forced equation

$$\frac{d^2y}{dt^2} + 2y = \cos \omega t$$

for ω very close to $\sqrt{2}$. As pointed out above, the frequency of the beats decreases and the amplitude of the forced response increases as ω approaches $\sqrt{2}$. A sequence of solutions with ω approaching $\sqrt{2}$ is shown in Figure 4.20.

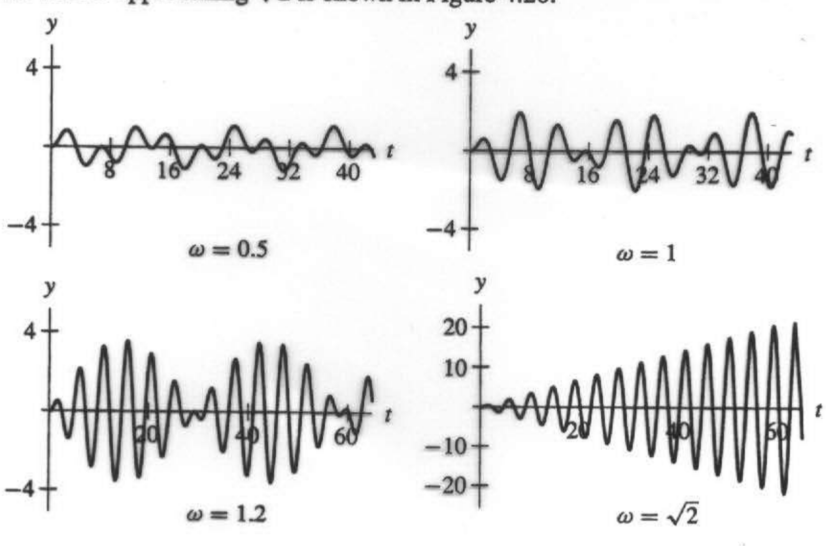

Figure 4.20
Solutions of $d^2y/dt^2 + 2y = \cos \omega t$ with $y(0) = y'(0) = 0$ for $\omega = 0.5$, $\omega = 1$, $\omega = 1.2$, and $\omega = \sqrt{2}$.