

Fundamentals of Plane Geometry involving Straight-Lines

ΣΤΟΙΧΕΙΩΝ α'.

ELEMENTS BOOK 1

Ὅροι.

Definitions

"lines"



- $\alpha = 1$
- $\beta = 2$
- $\gamma = 3$
- $\delta = 4$
- $\epsilon = 5$
- $\varphi = 6$
- $\zeta = 7$
- $\eta = 8$
- $\theta = 9$
- $\iota = 10$

$\kappa\alpha = 11$

$\chi = 20$

- α'. Σημεῖόν ἐστιν, οὐ μέρος οὐθέν.
 β'. Γραμμὴ δὲ μῆκος ἀπλατές.
 γ'. Γραμμῆς δὲ πέρατα σημεῖα.
 δ'. Εὐθεῖα γραμμὴ ἐστίν, ἣτις ἐξ ἴσου τοῖς ἐφ' ἑαυτῆς σημεῖοις κεῖται.
 ε'. Ἐπιφάνεια δὲ ἐστίν, ἧ μῆκος καὶ πλάτος μόνον ἔχει.
 ς'. Ἐπιφανείας δὲ πέρατα γραμμαῖ.
 ζ'. Ἐπίπεδος ἐπιφάνειά ἐστίν, ἣτις ἐξ ἴσου ταῖς ἐφ' ἑαυτῆς εὐθειαῖς κεῖται.
 η'. Ἐπίπεδος δὲ γωνία ἐστίν ἢ ἐν ἐπιπέδῳ δύο γραμμῶν ἀπτομένῳ ἀλλήλαις καὶ μὴ ἐπ' εὐθείας κειμένων πρὸς ἀλλήλας τῶν γραμμῶν κλίσις.
 θ'. Ὄταν δὲ αἱ περιέχουσαι τὴν γωνίαν γραμμαὶ εὐθεῖαι ὦσιν, εὐθύγραμμος καλεῖται ἡ γωνία.
 ι'. Ὄταν δὲ εὐθεῖα ἐπ' εὐθείαν σταθεῖσα τὰς ἐφεξῆς γωνίας ἴσας ἀλλήλαις ποιῇ, ὀρθὴ ἑκατέρα τῶν ἴσων γωνιῶν ἐστί, καὶ ἡ ἐφεστηκυῖα εὐθεῖα κάθετος καλεῖται, ἐφ' ἣν ἐφέστηκεν.
 ια'. Ἀμβλεῖα γωνία ἐστίν ἢ μείζων ὀρθῆς.
 ιβ'. Ὀξεῖα δὲ ἢ ἐλάσσων ὀρθῆς.
 ιγ'. Ὄρος ἐστίν, ὃ τινός ἐστι πέρασ.
 ιδ'. Σχήμα ἐστὶ τὸ ὑπὸ τινος ἢ τινῶν ὄρων περιεχόμενον.
 ιε'. Κύκλος ἐστὶ σχῆμα ἐπίπεδον ὑπὸ μιᾶς γραμμῆς περιεχόμενον [ἢ καλεῖται περιφέρεια], πρὸς ἣν ἀφ' ἐνὸς σημείου τῶν ἐντὸς τοῦ σχήματος κειμένων πᾶσαι αἱ προσπίπτουσαι εὐθεῖαι [πρὸς τὴν τοῦ κύκλου περιφέρειαν] ἴσαι ἀλλήλαις εἰσίν.
 ις'. Κέντρον δὲ τοῦ κύκλου τὸ σημεῖον καλεῖται.
 ιζ'. Διάμετρος δὲ τοῦ κύκλου ἐστίν εὐθεῖα τις διὰ τοῦ κέντρου ἠγμένη καὶ περατουμένη ἐφ' ἑκάτερα τὰ μέρη ὑπὸ τῆς τοῦ κύκλου περιφέρειας, ἣτις καὶ δίχα τέμνει τὸν κύκλον.
 ιη'. Ἡμικύκλιον δὲ ἐστὶ τὸ περιεχόμενον σχῆμα ὑπὸ τῆς διαμέτρου καὶ τῆς ἀπολαμβανομένης ὑπ' αὐτῆς περιφέρειας. κέντρον δὲ τοῦ ἡμικυκλίου τὸ αὐτό, ὃ καὶ τοῦ κύκλου ἐστίν.
 ιθ'. Σχήματα εὐθύγραμμά ἐστί τὰ ὑπὸ εὐθειῶν περιεχόμενα, τρίπλευρα μὲν τὰ ὑπὸ τριῶν, τετράπλευρα δὲ τὰ ὑπὸ τεσσάρων, πολὺπλευρα δὲ τὰ ὑπὸ πλειόνων ἢ τεσσάρων εὐθειῶν περιεχόμενα.
 κ'. Τῶν δὲ τριπλευρῶν σχημάτων ἰσόπλευρον μὲν τρίγωνόν ἐστὶ τὸ τὰς τρεῖς ἴσας ἔχον πλευράς, ἰσοσκελὲς δὲ τὸ τὰς δύο μόνους ἴσας ἔχον πλευράς, σκαληνὸν δὲ τὸ τὰς τρεῖς ἀνίσους ἔχον πλευράς.
 κα' Ἐτι δὲ τῶν τριπλευρῶν σχημάτων ὀρθογώνιον μὲν τρίγωνόν ἐστὶ τὸ ἔχον ὀρθὴν γωνίαν, ἀμβλυγώνιον δὲ τὸ ἔχον ἀμβλεῖαν γωνίαν, ὀξυγώνιον δὲ τὸ τὰς τρεῖς ὀξεῖας ἔχον γωνίας.

1. A point is that of which there is no part.
2. And a line is a length without breadth.
3. And the extremities of a line are points.
4. A straight-line is (any) one which lies evenly with points on itself.
5. And a surface is that which has length and breadth only.
6. And the extremities of a surface are lines.
7. A plane surface is (any) one which lies evenly with the straight-lines on itself. *in a plane*
8. And a plane angle is the inclination of the lines to one another, when two lines in a plane meet one another, and are not lying in a straight-line.
9. And when the lines containing the angle are straight then the angle is called rectilinear.
10. And when a straight-line stood upon (another) straight-line makes adjacent angles (which are) equal to one another, each of the equal angles is a right-angle, and the former straight-line is called a perpendicular to that upon which it stands.
11. An obtuse angle is one greater than a right-angle.
12. And an acute angle (is) one less than a right-angle.
13. A boundary is that which is the extremity of something.
14. A figure is that which is contained by some boundary or boundaries.
15. A circle is a plane figure contained by a single line [which is called a circumference], (such that) all of the straight-lines radiating towards [the circumference] from one point amongst those lying inside the figure are equal to one another.
16. And the point is called the center of the circle.
17. And a diameter of the circle is any straight-line, being drawn through the center, and terminated in each direction by the circumference of the circle. (And) any such (straight-line) also cuts the circle in half.†
18. And a semi-circle is the figure contained by the diameter and the circumference cuts off by it. And the center of the semi-circle is the same (point) as (the center of) the circle. *polygons*
19. Rectilinear figures are those (figures) contained by straight-lines: trilateral figures being those contained by three straight-lines, quadrilateral by four, and multi-lateral by more than four.
20. And of the trilateral figures: an equilateral triangle is that having three equal sides, an isosceles (triangle) that having only two equal sides, and a scalene (triangle) that having three unequal sides.

κβ'. Τῶν δὲ τετραπλευρῶν σχημάτων τετράγωνον μὲν ἔστιν, ὃ ἰσόπλευρόν τε ἔστι καὶ ὀρθογώνιον, ἑτερόμηκες δέ, ὃ ὀρθογώνιον μὲν, οὐκ ἰσόπλευρον δέ, ῥόμβος δέ, ὃ ἰσόπλευρον μὲν, οὐκ ὀρθογώνιον δέ, ῥομβοειδὲς δὲ τὸ τὰς ἀπεναντίον πλευρὰς τε καὶ γωνίας ἴσας ἀλλήλαις ἔχον, ὃ οὔτε ἰσόπλευρόν ἐστιν οὔτε ὀρθογώνιον· τὰ δὲ παρὰ ταῦτα τετράπλευρα τραπέζια καλεῖσθω.

κγ'. Παράλληλοί εἰσιν εὐθεῖαι, αἵτινες ἐν τῷ αὐτῷ ἐπιπέδῳ οὔσαι καὶ ἐκβαλλόμεναι εἰς ἄπειρον ἐφ' ἑκάτερα τὰ μέρη ἐπὶ μηδέτερα συμπίπτουσιν ἀλλήλαις.

21. And further of the trilateral figures: a right-angled triangle is that having a right-angle, an obtuse-angled (triangle) that having an obtuse angle, and an acute-angled (triangle) that having three acute angles.

22. And of the quadrilateral figures: a square is that which is right-angled and equilateral, a rectangle that which is right-angled but not equilateral, a rhombus that which is equilateral but not right-angled, and a rhomboid that having opposite sides and angles equal to one another which is neither right-angled nor equilateral. And let quadrilateral figures besides these be called trapezia.

23. Parallel lines are straight-lines which, being in the same plane, and being produced to infinity in each direction, meet with one another in neither (of these directions).

† This should really be counted as a postulate, rather than as part of a definition.

Αἰτήματα.

α'. Ἦιτήσθω ἀπὸ παντὸς σημείου ἐπὶ πᾶν σημῖον εὐθεῖαν γραμμὴν ἀγαγεῖν.

β'. Καὶ πεπερασμένην εὐθεῖαν κατὰ τὸ συνεχὲς ἐπ' εὐθείας ἐκβαλεῖν.

γ'. Καὶ παντὶ κέντρῳ καὶ διαστήματι κύκλον γράψεσθαι.

δ'. Καὶ πάσας τὰς ὀρθὰς γωνίας ἴσας ἀλλήλαις εἶναι.

ε'. Καὶ ἐὰν εἰς δύο εὐθείας εὐθεῖα ἐμπίπτουσα τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη γωνίας δύο ὀρθῶν ἐλάσσονας ποιῇ, ἐκβαλλομένας τὰς δύο εὐθείας ἐπ' ἄπειρον συμπίπτειν, ἐφ' ἃ μέρη εἰσὶν αἱ τῶν δύο ὀρθῶν ἐλάσσονες.



Postulates

1. Let it have been postulated† to draw a straight-line from any point to any point.

2. And to produce a finite straight-line continuously in a straight-line.

3. And to draw a circle with any center and radius.

4. And that all right-angles are equal to one another.

5. And that if a straight-line falling across two (other) straight-lines makes internal angles on the same side (of itself whose sum is) less than two right-angles, then the two (other) straight-lines, being produced to infinity, meet on that side (of the original straight-line) that the (sum of the internal angles) is less than two right-angles (and do not meet on the other side).‡

† The Greek present perfect tense indicates a past action with present significance. Hence, the 3rd-person present perfect imperative Ἦιτήσθω could be translated as "let it be postulated", in the sense "let it stand as postulated", but not "let the postulate be now brought forward". The literal translation "let it have been postulated" sounds awkward in English, but more accurately captures the meaning of the Greek.

‡ This postulate effectively specifies that we are dealing with the geometry of flat, rather than curved, space.

Κοινὰ ἔννοιαι.

α'. Τὰ τῷ αὐτῷ ἴσα καὶ ἀλλήλοις ἔστιν ἴσα.

β'. Καὶ ἐὰν ἴσοις ἴσα προστεθῇ, τὰ ὅλα ἔστιν ἴσα.

γ'. Καὶ ἐὰν ἀπὸ ἴσων ἴσα ἀφαιρεθῇ, τὰ καταλειπόμενά ἐστιν ἴσα.

δ'. Καὶ τὰ ἐφαρμόζοντα ἐπ' ἀλλήλα ἴσα ἀλλήλοις ἔστιν.

ε'. Καὶ τὸ ὅλον τοῦ μέρους μεῖζόν [ἔστιν].

Common Notions

(What is "equal"?)

1. Things equal to the same thing are also equal to one another.

2. And if equal things are added to equal things then the wholes are equal.

3. And if equal things are subtracted from equal things then the remainders are equal.†

4. And things coinciding with one another are equal to one another.

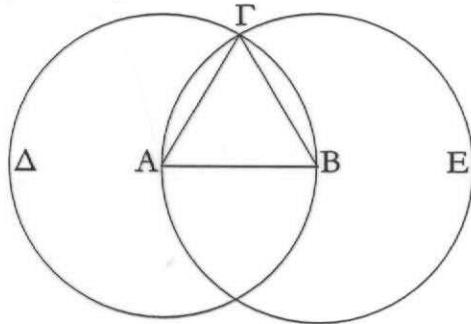
5. And the whole [is] greater than the part.

† As an obvious extension of C.N.s 2 & 3—if equal things are added or subtracted from the two sides of an inequality then the inequality remains

an inequality of the same type.

α'.

Ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης τριγώνων ἰσόπλευρον συστήσασθαι.



Ἐστω ἡ δοθείσα εὐθεῖα πεπερασμένη ἡ AB. Δεῖ δὴ ἐπὶ τῆς AB εὐθείας τριγώνων ἰσόπλευρον συστήσασθαι.

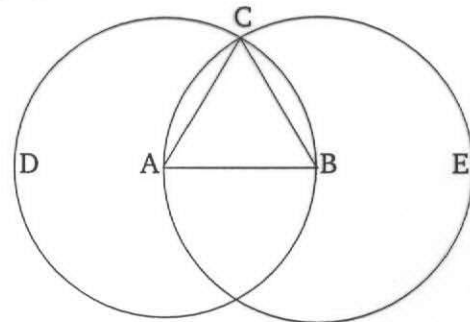
Κέντρῳ μὲν τῷ A διαστήματι δὲ τῷ AB κύκλος γεγράφθω ὁ BΓΔ, καὶ πάλιν κέντρῳ μὲν τῷ B διαστήματι δὲ τῷ BA κύκλος γεγράφθω ὁ AΓΕ, καὶ ἀπὸ τοῦ Γ σημείου, καθ' ὃ τέμνουσιν ἀλλήλους οἱ κύκλοι, ἐπὶ τὰ A, B σημεία ἐπεζεύχθωσαν εὐθεῖαι αἱ ΓΑ, ΓΒ.

Καὶ ἐπεὶ τὸ A σημεῖον κέντρον ἐστὶ τοῦ ΓΔB κύκλου, ἴση ἐστὶν ἡ AΓ τῇ AB· πάλιν, ἐπεὶ τὸ B σημεῖον κέντρον ἐστὶ τοῦ ΓAE κύκλου, ἴση ἐστὶν ἡ BΓ τῇ BA. ἐδείχθη δὲ καὶ ἡ ΓΑ τῇ AB ἴση· ἑκατέρα ἄρα τῶν ΓΑ, ΓΒ τῇ AB ἐστὶν ἴση. τὰ δὲ τῷ αὐτῷ ἴσα καὶ ἀλλήλοις ἐστὶν ἴσα· καὶ ἡ ΓΑ ἄρα τῇ ΓΒ ἐστὶν ἴση· αἱ τρεῖς ἄρα αἱ ΓΑ, AB, BΓ ἴσαι ἀλλήλαις εἰσίν.

Ἰσόπλευρον ἄρα ἐστὶ τὸ ABΓ τρίγωνον. καὶ συνέσταται ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης τῆς AB. ὅπερ ἔδει ποιῆσαι.

Proposition 1

To construct an equilateral triangle on a given finite straight-line.



Let AB be the given finite straight-line.

So it is required to construct an equilateral triangle on the straight-line AB .

Let the circle BCD with center A and radius AB have been drawn [Post. 3], and again let the circle ACE with center B and radius BA have been drawn [Post. 3]. And let the straight-lines CA and CB have been joined from the point C , where the circles cut one another,† to the points A and B (respectively) [Post. 1].

And since the point A is the center of the circle CDB , AC is equal to AB [Def. 1.15]. Again, since the point B is the center of the circle CAE , BC is equal to BA [Def. 1.15]. But CA was also shown (to be) equal to AB . Thus, CA and CB are each equal to AB . But things equal to the same thing are also equal to one another [C.N. 1]. Thus, CA is also equal to CB . Thus, the three (straight-lines) CA , AB , and BC are equal to one another.

Thus, the triangle ABC is equilateral, and has been constructed on the given finite straight-line AB . (Which is) the very thing it was required to do.

† The assumption that the circles do indeed cut one another should be counted as an additional postulate. There is also an implicit assumption that two straight-lines cannot share a common segment.

β'.

Πρὸς τῷ δοθέντι σημείῳ τῇ δοθείσῃ εὐθείᾳ ἴσην εὐθείαν θέσθαι.

Ἐστω τὸ μὲν δοθὲν σημεῖον τὸ A, ἡ δὲ δοθείσα εὐθεῖα ἡ BΓ· δεῖ δὴ πρὸς τῷ A σημείῳ τῇ δοθείσῃ εὐθείᾳ τῇ BΓ ἴσην εὐθείαν θέσθαι.

Ἐπεζεύχθω γὰρ ἀπὸ τοῦ A σημείου ἐπὶ τὸ B σημεῖον εὐθεῖα ἡ AB, καὶ συνεστάτω ἐπ' αὐτῆς τριγώνων ἰσόπλευρον τὸ ΔAB, καὶ ἐκβεβλήσθωσαν ἐπ' εὐθείας ταῖς ΔA, ΔB

Proposition 2†

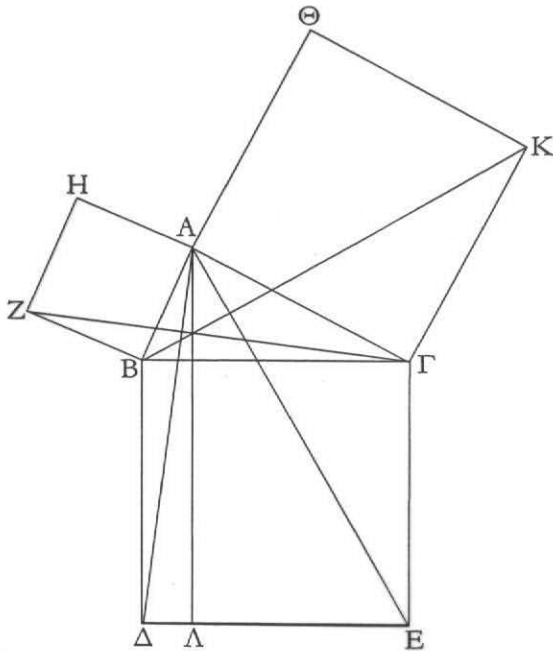
To place a straight-line equal to a given straight-line at a given point (as an extremity).

Let A be the given point, and BC the given straight-line. So it is required to place a straight-line at point A equal to the given straight-line BC .

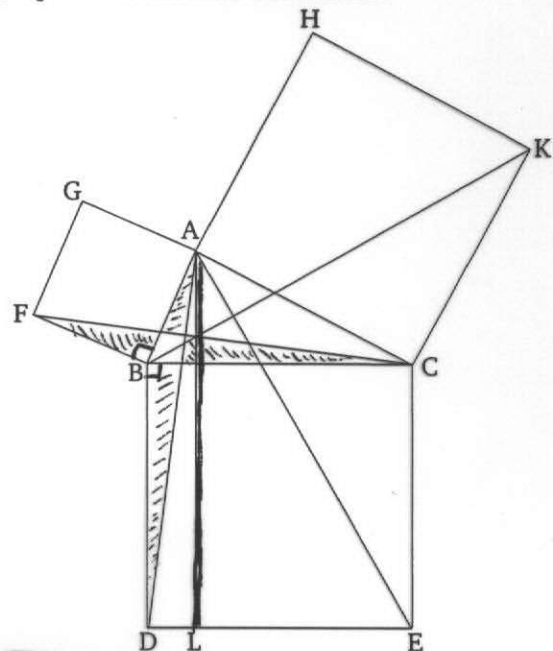
For let the straight-line AB have been joined from point A to point B [Post. 1], and let the equilateral triangle DAB have been constructed upon it [Prop. 1.1].

τρίγωνον τῷ ΖΒΓ τριγώνῳ ἔστιν ἴσον· καὶ [ἔστι] τοῦ μὲν ΑΒΔ τριγώνου διπλάσιον τὸ ΒΑ παραλληλόγραμμον· βάσιν τε γὰρ τὴν αὐτὴν ἔχουσι τὴν ΒΔ καὶ ἐν ταῖς αὐταῖς εἰσι παραλλήλοις ταῖς ΒΔ, ΑΛ· τοῦ δὲ ΖΒΓ τριγώνου διπλάσιον τὸ ΗΒ τετράγωνον· βάσιν τε γὰρ πάλιν τὴν αὐτὴν ἔχουσι τὴν ΖΒ καὶ ἐν ταῖς αὐταῖς εἰσι παραλλήλοις ταῖς ΖΒ, ΗΓ. [τὰ δὲ τῶν ἴσων διπλάσια ἴσα ἀλλήλοις ἔστιν·] ἴσον ἄρα ἔστι καὶ τὸ ΒΑ παραλληλόγραμμον τῷ ΗΒ τετραγώνῳ. ὁμοίως δὴ ἐπιζευγνυμένων τῶν ΑΕ, ΒΚ δειχθήσεται καὶ τὸ ΓΑ παραλληλόγραμμον ἴσον τῷ ΘΓ τετραγώνῳ· ὅλον ἄρα τὸ ΒΔΕΓ τετράγωνον δυοῖς τοῖς ΗΒ, ΘΓ τετραγώνοις ἴσον ἔστιν. καὶ ἔστι τὸ μὲν ΒΔΕΓ τετράγωνον ἀπὸ τῆς ΒΓ ἀναγραφέν, τὰ δὲ ΗΒ, ΘΓ ἀπὸ τῶν ΒΑ, ΑΓ. τὸ ἄρα ἀπὸ τῆς ΒΓ πλευρᾶς τετράγωνον ἴσον ἔστι τοῖς ἀπὸ τῶν ΒΑ, ΑΓ πλευρῶν τετραγώνοις.

two (straight-lines) CB, BF ,[†] respectively. And angle DBA (is) equal to angle FBC . Thus, the base AD [is] equal to the base FC , and the triangle ABD is equal to the triangle FBC [Prop. 1.4]. And parallelogram BL [is] double (the area) of triangle ABD . For they have the same base, BD , and are between the same parallels, BD and AL [Prop. 1.41]. And square GB is double (the area) of triangle FBC . For again they have the same base, FB , and are between the same parallels, FB and GC [Prop. 1.41]. [And the doubles of equal things are equal to one another.][‡] Thus, the parallelogram BL is also equal to the square GB . So, similarly, AE and BK being joined, the parallelogram CL can be shown (to be) equal to the square HC . Thus, the whole square $BDEC$ is equal to the (sum of the) two squares GB and HC . And the square $BDEC$ is described on BC , and the (squares) GB and HC on BA and AC (respectively). Thus, the square on the side BC is equal to the (sum of the) squares on the sides BA and AC .



Ἐν ἄρα τοῖς ὀρθογωνίοις τριγώνοις τὸ ἀπὸ τῆς τὴν ὀρθὴν γωνίαν ὑποτείνουσας πλευρᾶς τετράγωνον ἴσον ἔστι τοῖς ἀπὸ τῶν τὴν ὀρθὴν [γωνίαν] περιεχουσῶν πλευρῶν τετραγώνοις· ὅπερ ἔδει δεῖξαι.



Thus, in right-angled triangles, the square on the side subtending the right-angle is equal to the (sum of the) squares on the sides surrounding the right-[angle]. (Which is) the very thing it was required to show.

[†] The Greek text has " FB, BC ", which is obviously a mistake.

[‡] This is an additional common notion.

Fundamentals of Plane Geometry Involving Circles

τὸ AEB τμήμα ἐπὶ τὸ $\Gamma Z \Delta$ · ἐφαρμόσει ἄρα, καὶ ἴσον αὐτῷ ἔσται.

Τὰ ἄρα ἐπὶ ἴσων εὐθειῶν ὅμοια τμήματα κύκλων ἴσα ἀλλήλοις ἐστίν· ὅπερ ἔδει δεῖξαι.

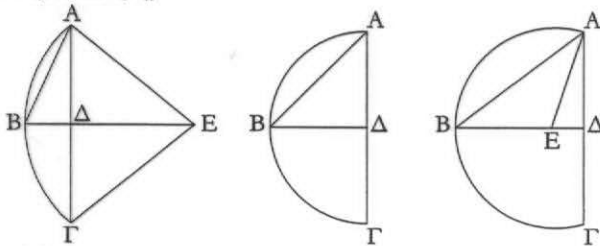
thing is impossible [Prop. 3.10]. Thus, if the straight-line AB is applied to CD , the segment AEB cannot not also coincide with CFD . Thus, it will coincide, and will be equal to it [C.N. 4].

Thus, similar segments of circles on equal straight-lines are equal to one another. (Which is) the very thing it was required to show.

† Both this possibility, and the previous one, are precluded by Prop. 3.23.

κε΄.

Κύκλου τμήματος δοθέντος προσαναγράψαι τὸν κύκλον, οὐπὲρ ἔστι τμήμα.



Ἐστω τὸ δοθὲν τμήμα κύκλου τὸ $AB\Gamma$ · δεῖ δὴ τοῦ $AB\Gamma$ τμήματος προσαναγράψαι τὸν κύκλον, οὐπὲρ ἔστι τμήμα.

Τετμήσθω γὰρ ἡ AG δίχα κατὰ τὸ Δ , καὶ ἤχθω ἀπὸ τοῦ Δ σημείου τῇ AG πρὸς ὀρθὰς ἡ ΔB , καὶ ἐπεζεύχθω ἡ AB · ἡ ὑπὸ $AB\Delta$ γωνία ἄρα τῆς ὑπὸ BAD ἥτοι μείζων ἐστίν ἢ ἴση ἢ ἐλάττων.

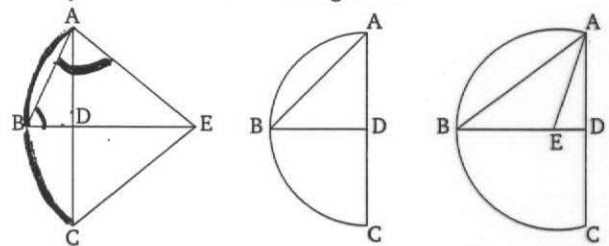
Ἐστω πρότερον μείζων, καὶ συνεστάτω πρὸς τῇ BA εὐθείᾳ καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ A τῇ ὑπὸ $AB\Delta$ γωνίᾳ ἴση ἢ ὑπὸ BAE , καὶ διήχθω ἡ ΔB ἐπὶ τὸ E , καὶ ἐπεζεύχθω ἡ EF . ἐπεὶ οὖν ἴση ἐστὶν ἡ ὑπὸ ABE γωνία τῇ ὑπὸ BAE , ἴση ἄρα ἐστὶ καὶ ἡ EB εὐθεῖα τῇ EA . καὶ ἐπεὶ ἴση ἐστὶν ἡ AD τῇ $\Delta\Gamma$, κοινὴ δὲ ἡ ΔE , δύο δὴ αἱ AD , ΔE δύο ταῖς $\Gamma\Delta$, ΔE ἴσαι εἰσὶν ἑκατέρωθεν· καὶ γωνία ἡ ὑπὸ $A\Delta E$ γωνία τῇ ὑπὸ $\Gamma\Delta E$ ἐστὶν ἴση· ὀρθὴ γὰρ ἑκατέρωθεν· βάσεις ἄρα ἢ AE βάσει τῇ ΓE ἐστὶν ἴση. ἀλλὰ ἡ AE τῇ BE ἐδείχθη ἴση· καὶ ἡ BE ἄρα τῇ ΓE ἐστὶν ἴση· αἱ τρεῖς ἄρα αἱ AE , EB , EF ἴσαι ἀλλήλαις εἰσὶν· ὁ ἄρα κέντρῳ τῷ E διαστήματι δὲ ἐνὶ τῶν AE , EB , EF κύκλος γραφόμενος ἤξει καὶ διὰ τῶν λοιπῶν σημείων καὶ ἔσται προσαναγεγραμμένος· κύκλου ἄρα τμήματος δοθέντος προσαναγράφεται ὁ κύκλος· καὶ δῆλον, ὡς τὸ $AB\Gamma$ τμήμα ἐλαττόν ἐστιν ἡμικυκλίου διὰ τὸ τὸ E κέντρον ἐκτὸς αὐτοῦ τυγχάνειν.

Ὅμοιως [δὲ] καὶ ἢ ἡ ὑπὸ $AB\Delta$ γωνία ἴση τῇ ὑπὸ BAD , τῆς $A\Delta$ ἴσης γενομένης ἑκατέρωθεν τῶν $B\Delta$, $\Delta\Gamma$ αἱ τρεῖς αἱ ΔA , ΔB , $\Delta\Gamma$ ἴσαι ἀλλήλαις ἔσονται, καὶ ἔσται τὸ Δ κέντρον τοῦ προσαναπεληρωμένου κύκλου, καὶ δηλαδὴ ἔσται τὸ $AB\Gamma$ ἡμικύκλιον.

Ἐὰν δὲ ἡ ὑπὸ $AB\Delta$ ἐλάττων ἢ τῆς ὑπὸ BAD , καὶ συστησώμεθα πρὸς τῇ BA εὐθείᾳ καὶ τῷ πρὸς αὐτῇ σημείῳ

Proposition 25

For a given segment of a circle, to complete the circle, the very one of which it is a segment.



Let ABC be the given segment of a circle. So it is required to complete the circle for segment ABC , the very one of which it is a segment.

For let AC have been cut in half at (point) D [Prop. 1.10], and let DB have been drawn from point D , at right-angles to AC [Prop. 1.11]. And let AB have been joined. Thus, angle ABD is surely either greater than, equal to, or less than (angle) BAD .

First of all, let it be greater. And let (angle) BAE , equal to angle ABD , have been constructed on the straight-line BA , at the point A on it [Prop. 1.23]. And let DB have been drawn through to E , and let EC have been joined. Therefore, since angle ABE is equal to BAE , the straight-line EB is thus also equal to EA [Prop. 1.6]. And since AD is equal to DC , and DE (is) common, the two (straight-lines) AD , DE are equal to the two (straight-lines) CD , DE , respectively. And angle ADE is equal to angle CDE . For each (is) a right-angle. Thus, the base AE is equal to the base CE [Prop. 1.4]. But, AE was shown (to be) equal to BE . Thus, BE is also equal to CE . Thus, the three (straight-lines) AE , EB , and EC are equal to one another. Thus, if a circle is drawn with center E , and radius one of AE , EB , or EC , it will also go through the remaining points (of the segment), and the (associated circle) will have been completed [Prop. 3.9]. Thus, a circle has been completed from the given segment of a circle. And (it is) clear that the segment ABC is less than a semi-circle, because the center E happens to lie outside it.

Case 1

φερείας περιεχομένη ἐλάττων ἐστὶν ὀρθῆς.

Ἐν κύκλῳ ἄρα ἡ μὲν ἐν τῷ ἡμικυκλίῳ γωνία ὀρθή ἐστίν, ἡ δὲ ἐν τῷ μείζονι τμήματι ἐλάττων ὀρθῆς, ἡ δὲ ἐν τῷ ἐλάττονι [τμήματι] μείζων ὀρθῆς· καὶ ἔπι ἡ μὲν τοῦ μείζονος τμήματος [γωνία] μείζων [ἐστίν] ὀρθῆς, ἡ δὲ τοῦ ἐλάττονος τμήματος [γωνία] ἐλάττων ὀρθῆς· ὅπερ ἔδει δεῖξαι.

by the circumference $AD[C]$ and the straight-line AC , is less than a right-angle. And this is immediately apparent. For since the (angle contained by) the two straight-lines BA and AC is a right-angle, the (angle) contained by the circumference ABC and the straight-line AC is thus greater than a right-angle. Again, since the (angle contained by) the straight-lines AC and AF is a right-angle, the (angle) contained by the circumference $AD[C]$ and the straight-line CA is thus less than a right-angle.

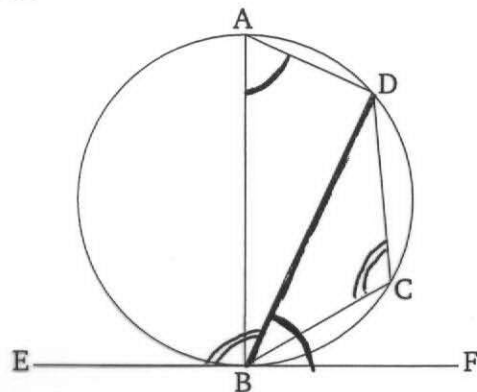
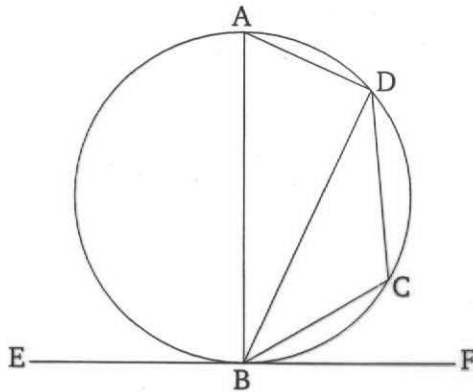
Thus, in a circle, the angle in a semi-circle is a right-angle, and that in a greater segment (is) less than a right-angle, and that in a lesser [segment] (is) greater than a right-angle. And, further, the [angle] of a segment greater (than a semi-circle) [is] greater than a right-angle, and the [angle] of a segment less (than a semi-circle) is less than a right-angle. (Which is) the very thing it was required to show.

λβ΄.

Ἐὰν κύκλου ἐφάπτηται τις εὐθεῖα, ἀπὸ δὲ τῆς ἀφῆς εἰς τὸν κύκλον διαχθῆ τις εὐθεῖα τέμνουσα τὸν κύκλον, ἄς ποιῆ γωνίας πρὸς τῇ ἐφαπτομένῃ, ἴσαι ἔσονται ταῖς ἐν τοῖς ἐναλλάξ τοῦ κύκλου τμήμασι γωνίαις.

Proposition 32

If some straight-line touches a circle, and some (other) straight-line is drawn across, from the point of contact into the circle, cutting the circle (in two), then those angles the (straight-line) makes with the tangent will be equal to the angles in the alternate segments of the circle.



Κύκλου γὰρ τοῦ $ABΓΔ$ ἐφαπτέσθω τις εὐθεῖα ἡ EZ κατὰ τὸ B σημεῖον, καὶ ἀπὸ τοῦ B σημείου διήχθω τις εὐθεῖα εἰς τὸν $ABΓΔ$ κύκλον τέμνουσα αὐτὸν ἡ BD . λέγω, ὅτι ἄς ποιῆ γωνίας ἡ BD μετὰ τῆς EZ ἐφαπτομένης, ἴσας ἔσονται ταῖς ἐν τοῖς ἐναλλάξ τμήμασι τοῦ κύκλου γωνίαις, τουτέστιν, ὅτι ἡ μὲν ὑπὸ ZBD γωνία ἴση ἐστὶ τῇ ἐν τῷ BAD τμήματι συνισταμένῃ γωνίᾳ, ἡ δὲ ὑπὸ EBD γωνία ἴση ἐστὶ τῇ ἐν τῷ $ΔGB$ τμήματι συνισταμένῃ γωνίᾳ.

For let some straight-line EF touch the circle $ABCD$ at the point B , and let some (other) straight-line BD have been drawn from point B into the circle $ABCD$, cutting it (in two). I say that the angles BD makes with the tangent EF will be equal to the angles in the alternate segments of the circle. That is to say, that angle FBD is equal to the angle constructed in segment BAD , and angle EBD is equal to the angle constructed in segment DCB .

Ἦχθω γὰρ ἀπὸ τοῦ B τῇ EZ πρὸς ὀρθὰς ἡ BA , καὶ εἰλήφθω ἐπὶ τῆς BD περιφερείας τυχὸν σημεῖον τὸ $Γ$, καὶ ἐπεζεύχθωσαν αἱ AD , $ΔΓ$, $ΓB$.

For let BA have been drawn from B , at right-angles to EF [Prop. 1.11]. And let the point C have been taken at random on the circumference BD . And let AD , DC ,

Καὶ ἐπεὶ κύκλου τοῦ $ABΓΔ$ ἐφάπτεται τις εὐθεῖα ἡ EZ

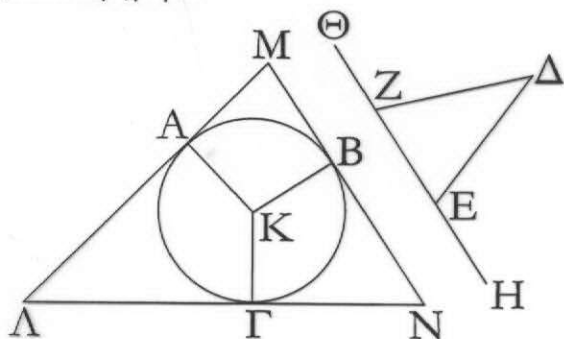
Construction of Rectilinear Figures (Polygons)

In and Around Circles

† See the footnote to Prop. 3.34.

Υ΄.

Περί τόν δοθέντα κύκλον τῷ δοθέντι τριγώνῳ ἰσογώνιον τρίγωνον περιγράψαι.



Ἐστω ὁ δοθείς κύκλος ὁ $AB\Gamma$, τὸ δὲ δοθὲν τρίγωνον τὸ ΔEZ . δεῖ δὴ περὶ τὸν $AB\Gamma$ κύκλον τῷ ΔEZ τριγώνῳ ἰσογώνιον τρίγωνον περιγράψαι.

Ἐκβεβλήσθω ἡ EZ ἐφ' ἐκάτερα τὰ μέρη κατὰ τὰ H, Θ σημεῖα, καὶ εἰλήφθω τοῦ $AB\Gamma$ κύκλου κέντρον τὸ K , καὶ διήχθω, ὡς ἔτυχεν, εὐθεῖα ἡ KB , καὶ συνεστάτω πρὸς τῇ KB εὐθείᾳ καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ K τῇ μὲν ὑπὸ ΔEH γωνία ἴση ἢ ὑπὸ BKA , τῇ δὲ ὑπὸ $\Delta Z\Theta$ ἴση ἢ ὑπὸ $BK\Gamma$, καὶ διὰ τῶν A, B, Γ σημείων ἤχθωσαν ἐφαπτόμεναι τοῦ $AB\Gamma$ κύκλου αἱ $\Lambda AM, MBN, N\Gamma A$.

Καὶ ἐπεὶ ἐφαπτόνται τοῦ $AB\Gamma$ κύκλου αἱ $\Lambda M, MN, N\Lambda$ κατὰ τὰ A, B, Γ σημεῖα, ἀπὸ δὲ τοῦ K κέντρου ἐπὶ τὰ A, B, Γ σημεῖα ἐπεζευγμέναι εἰσὶν αἱ $KA, KB, K\Gamma$, ὀρθαὶ ἄρα εἰσὶν αἱ πρὸς τοῖς A, B, Γ σημείοις γωνίαι. καὶ ἐπεὶ τοῦ $AMBK$ τετραπλεύρου αἱ τέσσαρες γωνίαι τέτρασιν ὀρθαῖς ἴσαι εἰσὶν, ἐπειδήπερ καὶ εἰς δύο τρίγωνα διαιρεῖται τὸ $AMBK$, καὶ εἰσὶν ὀρθαὶ αἱ ὑπὸ KAM, KBM γωνίαι, λοιπαὶ ἄρα αἱ ὑπὸ AKB, AMB δυσὶν ὀρθαῖς ἴσαι εἰσὶν. εἰσὶ δὲ καὶ αἱ ὑπὸ $\Delta EH, \Delta EZ$ δυσὶν ὀρθαῖς ἴσαι· αἱ ἄρα ὑπὸ AKB, AMB ταῖς ὑπὸ $\Delta EH, \Delta EZ$ ἴσαι εἰσὶν, ὧν ἡ ὑπὸ AKB τῇ ὑπὸ ΔEH ἔστιν ἴση· λοιπὴ ἄρα ἡ ὑπὸ AMB λοιπὴ τῇ ὑπὸ ΔEZ ἔστιν ἴση. ὁμοίως δὴ δειχθήσεται, ὅτι καὶ ἡ ὑπὸ ANB τῇ ὑπὸ ΔZE ἔστιν ἴση· καὶ λοιπὴ ἄρα ἡ ὑπὸ MAN [λοιπῆ] τῇ ὑπὸ ΔEZ ἔστιν ἴση. ἰσογώνιον ἄρα ἔστι τὸ ΛMN τρίγωνον τῷ ΔEZ τριγώνῳ· καὶ περιέγραπται περὶ τὸν $AB\Gamma$ κύκλον.

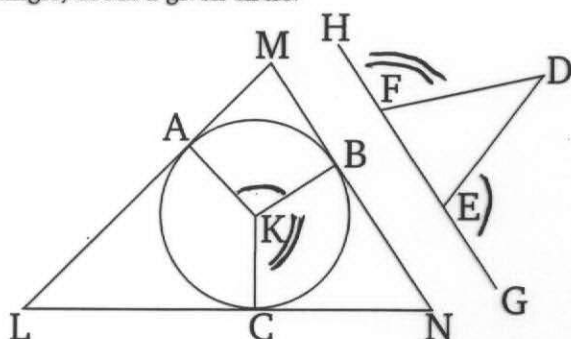
Περί τόν δοθέντα ἄρα κύκλον τῷ δοθέντι τριγώνῳ ἰσογώνιον τρίγωνον περιέγραπται· ὅπερ ἔδει ποιῆσαι.

ABC].

Thus, a triangle, equiangular with the given triangle, has been inscribed in the given circle. (Which is) the very thing it was required to do.

Proposition 3

To circumscribe a triangle, equiangular with a given triangle, about a given circle.



Let ABC be the given circle, and DEF the given triangle. So it is required to circumscribe a triangle, equiangular with triangle DEF , about circle ABC .

Let EF have been produced in each direction to points G and H . And let the center K of circle ABC have been found [Prop. 3.1]. And let the straight-line KB have been drawn, at random, across (ABC) . And let (angle) BKA , equal to angle DEG , have been constructed on the straight-line KB at the point K on it, and (angle) BKC , equal to DFH [Prop. 1.23]. And let the (straight-lines) LAM, MBN , and NCL have been drawn through the points A, B , and C (respectively), touching the circle ABC .†

And since LM, MN , and NL touch circle ABC at points A, B , and C (respectively), and KA, KB , and KC are joined from the center K to points A, B , and C (respectively), the angles at points A, B , and C are thus right-angles [Prop. 3.18]. And since the (sum of the) four angles of quadrilateral $AMBK$ is equal to four right-angles, inasmuch as $AMBK$ (can) also (be) divided into two triangles [Prop. 1.32], and angles KAM and KBM are (both) right-angles, the (sum of the) remaining (angles), AKB and AMB , is thus equal to two right-angles. And DEG and DEF is also equal to two right-angles [Prop. 1.13]. Thus, AKB and AMB is equal to DEG and DEF , of which AKB is equal to DEG . Thus, the remainder AMB is equal to the remainder DEF . So, similarly, it can be shown that LNB is also equal to DFE . Thus, the remaining (angle) MLN is also equal to the