

# Diophantus (200-284 AD?)

$$\begin{matrix} b=20 \\ c=208 \end{matrix}$$

I-28: Given  $b, c$  find  $x, y$  so that  $x+y=b, x^2+y^2=c$

Babylonian-style problem, and solution

Ignored negative and irrational solutions.

$$2c = b^2 + d^2 \geq b^2$$

IV-29: Find  $a, b, c, d$  so that  $(a^2+b^2+c^2+d^2) + (a+b+c+d) = K$

Solution:  $x^2 + x + 1/4 = (x + 1/2)^2$  so need  $x_1^2 + x_2^2 + x_3^2 + x_4^2 = K+1$

Given  $K=12, 13 = 4+9 = (\frac{8}{5})^2 + (\frac{6}{5})^2 + (\frac{12}{5})^2 + (\frac{9}{5})^2$

B-7 Given  $b, c$  find  $x, y$  so that  $x+y=b, x^3+y^3=c$

Fix  $b=20, c=2240$  start with  $\frac{b}{2} = 10$

$$(10-\delta)^3 + (10+\delta)^3 = 2000 + 60\delta^2 = 2240$$

$$\begin{matrix} \frac{b}{2} + \frac{c}{b^2} = 6 \\ \frac{b}{2}^3 + \frac{c}{b^2} = 2000 \approx c \end{matrix}$$

$$60\delta^2 = 240$$

$$\delta^2 = 4$$

$$\delta = 2$$

$$\begin{matrix} x = 10 - \delta = 8 \\ y = 10 + \delta = 12 \end{matrix}$$

A-25 Find  $x, y, z$  so that  $(x^2)^2 + (y^3)^2 = z^2$

Take  $\begin{cases} x = ay \\ z = ky^2 \end{cases}$ , solve  $\begin{matrix} a^4 y^4 + y^6 = k^2 y^4 \\ a^4 + y^2 = k^2 \end{matrix}$

Already solved for all  $a$   $\left[ \begin{matrix} \text{If } a=2, \text{ get } y=3 \\ k=5 \quad x=6 \end{matrix} \right]$

Ὀτός τοι Διόφαντος ἔχει τάφος ἄ μέγα θαῦμα  
καὶ τάφος ἐκ τέχνης μέτρα βίοιο λέγει.  
ἕκτην κουρίζειν βίοτου θεὸς ὥπασε μοίρην  
δωδεκάτην δ' ἐπιθείς, μήλα πόρεν χροαίην  
τῇ δ' ἄρ' ἐφ' ἑβδομάτῃ τὸ γαμήλιον ἤψατο φέγγος, ἔ  
ἐκ δὲ γάμων πέμπτην παῖδ' ἐπένευσεν ἔτει.  
αἰαί, τηλύγετον δειλὸν τέκος, ἦμισυ πατρὸς  
† τοῦδε καὶ ἡ κρυερὸς μέτρον ἑλὼν βίοτου.  
πένθος δ' αὐτὸν πιαύρεσσι παρηγορέων ἐνιαυτοῖς  
τῆδε πόσον σοφίῃ τέρα' ἐπέρησε βίου.

This tomb holds Diophantus. Ah, how great a marvel! the tomb tells scientifically the measure of his life. God granted him to be a boy for the sixth part of his life, and adding a twelfth part to this, he clothed his cheeks with down; He lit him the light of wedlock after a seventh part, and five years after his marriage He granted him a son. Alas! late-born wretched child; after attaining the measure of half his father's life, chill Fate took him. After consoling his grief by this science of numbers for four years he ended his life.